

TRANSPOSISI DIDAKTIK BERPIKIR ALJABAR AWAL

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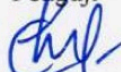
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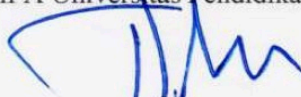


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KATA PENGANTAR

Puji dan syukur penulis panjatkan ke hadirat Allah Subhanahu Wa Ta'ala, karena atas rahmat dan ridho-Nya penulis dapat menyelesaikan disertasi ini. Penulisan disertasi ini dalam rangka memenuhi salah satu syarat dalam memperoleh gelar doktor pendidikan pada Program Studi Pendidikan Matematika Fakultas Pendidikan Matematika dan Ilmu Pengetahuan Alam Universitas Pendidikan Indonesia.

Disertasi yang berjudul “Transposisi Didaktik Berpikir Aljabar Awal” ini bertujuan untuk mengkonstruksi makna di balik pengalaman belajar siswa terhadap munculnya pemikiran aljabar awal sebagai implementasi desain didaktik yang merupakan perwujudan dari *knowledge to be taught* yang dirancang peneliti setelah melalui proses transposisi didaktik. Peneliti ingin ikut serta berkontribusi dalam komunitas dunia yang berfokus pada berpikir aljabar awal. Peneliti ingin menghadirkan konteks Indonesia dalam penelitian berpikir aljabar awal. Dengan melakukan transposisi didaktik, peneliti membentuk pengetahuan esensial aljabar, panduan *knowledge to be taught*, dan desain didaktik berpikir aljabar awal. Dengan melakukan implementasi desain didaktik, peneliti menelusuri makna di balik pengalaman belajar siswa sekolah dasar yang dapat membentuk pemikiran aljabar awal. Peneliti berusaha meletakkan gagasan aljabar di konten matematika sekolah dasar, sehingga bisa menjadi rekomendasi bagi kurikulum sekolah dasar di Indonesia untuk mengakomodasi pemikiran aljabar.

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TRANSPOSISI DIDAKTIK BERPIKIR ALJABAR AWAL

ABSTRAK

Dalam pembelajaran aljabar, pengetahuan selalu berupa makna dan pemaknaannya terdapat di dalam proses transposisi sehingga bersifat situasional dan berupa abstraksi dari konteks penggunaannya. Penelitian ini bertujuan untuk mengkonstruksi makna di balik pengalaman belajar siswa terhadap munculnya pemikiran aljabar awal sebagai implementasi desain didaktik yang merupakan perwujudan dari *knowledge to be taught* yang dirancang peneliti setelah melalui proses transposisi didaktik. Metode penelitian yang digunakan adalah penelitian kualitatif dengan pendekatan fenomenologi hermeneutik. Objek matematis dikonstruksi dalam bentuk formal melalui proses re-depersonalisasi dan re-dekontekstualisasi, maka terbentuklah konstruksi *scholarly knowledge* mengenai aljabar esensial yang akan berkaitan dengan berpikir aljabar awal. *Scholarly knowledge* bersifat sebagai pengetahuan *a priori* sehingga perlu dilakukan transposisi menjadi materi kurikulum (*knowledge to be taught*). Transposisi diawali dengan repersonalisasi dan rekontekstualisasi sehingga menjadi pengetahuan yang bersifat *a posteriori*. Dalam proses transposisinya, peneliti juga melakukan analisis terhadap kurikulum serta buku ajar yang digunakan di sekolah dasar yang diteliti dengan menggunakan praxiologi. Peneliti membentuk pengetahuan esensial untuk diajarkan dan panduan *knowledge to be taught* di sekolah dasar yang dapat mengakomodasi berpikir aljabar awal sebagai pengalaman belajar siswa. Peneliti mengerucutkan permasalahan berpikir aljabar awal menjadi konsep spesifik proporsionalitas sebagai salah satu contoh dari pemikiran aljabar awal. Kurikulum matematika sekolah dasar ditransposisikan menjadi objek pengajaran di kelas dalam kerangka proses belajar mengajar (*taught knowledge*). Dalam proses transposisi untuk menyusun objek pengajaran, peneliti juga melakukan analisis terhadap proses pembelajaran matematika di sekolah dasar untuk mengidentifikasi beragam masalah yang terjadi, dan melakukan analisis *learning obstacle* konsep proporsionalitas di sekolah menengah. Peneliti menyusun objek pengajaran dalam bentuk desain didaktik yang terdiri dari dua komponen, yaitu praxis (*type of task* dan teknik) serta logos (teknologi dan teori). Dari implementasi desain didaktik, peneliti menganalisis pemikiran aljabar awal siswa yang terbentuk (*learnt knowledge*) dalam permasalahan proporsionalitas di sekolah dasar, antara lain melalui pendekatan tabel rasio dengan penalaran aditif dan multiplikatif, pendekatan rasio satuan, pendekatan faktor perubahan, pendekatan pecahan dan rasio yang ekuivalen, serta koneksi pendekatan rasio satuan dan faktor perubahan ke dalam algoritma standar. Sehingga terbentuk penalaran proporsional yang sebenarnya membangun berpikir aljabar awal mengenai konsep invarian dan kovarian, kelas ekuivalensi, dan fungsi linier.

Kata Kunci: Transposisi Didaktik, Berpikir Aljabar Awal, Proporsionalitas

DIDACTIC TRANSPOSITION OF EARLY ALGEBRA THINKING

ABSTRACT

In learning algebra, knowledge is always in the form of meaning and its meaning is contained in the transposition process so that it is situational and in the form of an abstraction from the context of its use. This study aims to construct the meaning behind the students' learning experience towards the emergence of early algebraic thinking as the implementation of a didactic design which is an embodiment of knowledge to be taught that was designed by researchers after going through a didactic transposition process. The research method used is qualitative research with a hermeneutic phenomenological approach. Mathematical objects are constructed in a formal form through the process of re-personalization and re-decontextualization, so that the construction of scholarly knowledge regarding essential algebra is formed which will be related to initial algebraic thinking. Scholarly knowledge is a priori knowledge so it is necessary to transpose it into curriculum material (knowledge to be taught). Transposition begins with re-personalization and recontextualization so that it becomes a posteriori knowledge. In the transposition process, the researcher also conducted an analysis of the curriculum and textbooks used in the elementary schools studied using praxiology. Researchers form essential knowledge to be taught and guide knowledge to be taught in elementary schools that can accommodate early algebraic thinking as a student learning experience. The researcher narrows the problem of early algebraic thinking into a specific concept of proportionality as an example of early algebraic thinking. The elementary school mathematics curriculum is transposed into teaching objects in class within the framework of the teaching and learning process (taught knowledge). In the process of transposition to arrange teaching objects, the researcher also conducted an analysis of the process of learning mathematics in elementary schools to identify various problems that occur, and conducted an analysis of learning obstacles to the concept of proportionality in secondary schools. The researcher arranges the teaching object in the form of a didactic design which consists of two components, namely praxis (type of task and technique) and logos (technology and theory). From the implementation of the didactic design, the researcher analyzed the students' initial algebraic thinking that was formed (learnt knowledge) in proportionality problems in elementary schools, including through the ratio table approach with additive and multiplicative reasoning, unit ratio approach, change factor approach, fractional approach and equivalent ratios, as well as the connection of unit ratio approaches and factor of change into standard algorithms. So that proportional reasoning is formed which actually builds initial algebraic thinking about the concepts of invariant and covariance, equivalence classes, and linear functions.

Keywords: Didactic Transposition, Early Algebraic Thinking, Proportionality

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GLOSARIUM

A

- A posteriori : Pengetahuan yang diperoleh setelah adanya pengalaman.
- A priori : Pengetahuan yang ada sebelum bertemu dengan pengalaman.
- Aljabar : Ilmu matematika dimana generalitas diekspresikan oleh simbol alfanumerik.
- Aljabar Awal : Pengenalan awal simbol untuk mengekspresikan pemikiran relasional dan koordinasi berbagai representasi.

B

- Berpikir Aljabar : Aktivitas mental seperti penalaran dengan variabel, menggeneralisasi dan memformalkan hubungan antara besaran-besaran dan pengembangan konsep variabel.
- Berpikir Aljabar Awal : Aktivitas mental yang dilakukan siswa ketika melihat hubungan antar objek (bilangan, kuantitas) dan kemudian mencoba untuk mengekspresikannya serta membangun makna untuk objek tersebut.

D

- Dekontekstualisasi : Proses progresif di mana matematika diekstraksi dari situasi masalah/konteks, sehingga menghasilkan matematika yang sesungguhnya, (tingkat pemikiran yang abstrak).
- Depersonalisasi : Praktik didaktis yang terdiri dari mengekstraksi pengalaman dan menempatkan pengetahuan ke dalam bentuk yang umum.
- Desain didaktik : Bahan ajar yang dirancang guru untuk membangun konsep serta meminimalisir hambatan belajar yang terjadi.
- Didaktik : Pengetahuan sistemik terkait difusi dan akuisisi pengetahuan dalam masyarakat.

I

- Invarian : Sifat dari objek matematika yang tetap tidak berubah setelah operasi atau transformasi tertentu diterapkan pada objek.

K

- Knowledge to be taught : Pengetahuan yang akan diajarkan yang berupa

		materi kurikulum dan buku ajar.
Kontekstualisasi	:	Membangun kasus-kasus khusus, untuk melihat khusus pada umumnya, untuk bergerak ke arah yang konkret dalam situasi representasional baru.
Kovarian	:	Ukuran untuk melihat bagaimana perubahan dalam suatu objek dikaitkan dengan perubahan dalam objek kedua.
L		
Learning obstacle	:	Hambatan belajar yang dialami siswa dalam konsep matematika tertentu.
Learnt knowledge	:	Pengetahuan yang dipelajari atau dipahami oleh siswa.
Logos	:	Blok teoritikal dalam praksiologi.
P		
Pecahan	:	Kelas ekuivalen dari pasangan bilangan bulat terurut yang memenuhi aksioma ekuivalensi, urutan, dan operasi tertentu.
Personalisasi	:	Proses mengkonstruksi objek matematika dengan pengalaman personal tertentu.
Praxiologi	:	Sebuah epistemologi model untuk memodelkan dan menganalisa pengetahuan matematika dan didaktik.
Praxis	:	Blok praktikal dalam praxiologi.
Proporsionalitas	:	Struktur matematis yang memodelkan hubungan fungsi linier $y = mx$ mengatur kovariansi antara dua kuantitas dalam situasi proporsional. dimana dua besaran/kuantitas, x dan y , berubah bersama sedemikian sehingga rasio antara besaran tetap sama.
R		
Rasio	:	Kelas ekuivalen dari barisan berurutan bilangan real yang memenuhi aksioma 1R dan 2R. Aksioma 1R. Tidak ada elemen nol dalam urutan. Aksioma 2R. (Silang - Aturan Perkalian) Dua barisan (a_1, a_2, \dots, a_n) dan (b_1, b_2, \dots, b_n) adalah ekuivalen: $(a_1, a_2, \dots, a_n) \sim (b_1, b_2, \dots, b_n)$ jika $a_i b_j = a_j b_i$, untuk semua $1 \leq i, j \leq n$.
Redekontekstualisasi	:	Proses mengkonstruksi kembali objek matematika ke dalam bentuk yang umum dan formal tanpa adanya konteks yang terkait.

Redepersonalisasi	: Proses mengkonstruksi kembali objek matematika ke dalam bentuk yang umum dan formal tanpa adanya terikat pengalaman.
Rekontekstualisasi	: Tindakan semiotik yang memungkinkan konfigurasi representasional, dengan mencari situasi yang dapat memberi makna pada pengetahuan yang akan diajarkan domain semantik baru.
Repersonalisasi	: Menghidupkan pengetahuan kembali dengan pengalaman personal yang dapat memberi makna pada pengetahuan yang akan diajarkan.
S	
Scholarly knowledge	: Pengetahuan yang berlaku secara umum atau bersifat a priori, yang dikembangkan oleh ilmunan/matematikawan.
T	
Taught knowledge	: Pengetahuan yang diajarkan guru di kelas selama proses pembelajaran.
Teknik	: Istilah dalam praxiologi untuk cara atau beberapa cara yang diperlukan untuk menyelesaikan masalah tersebut
Teknologi	: Istilah dalam praxiologi untuk argumentasi atau penjelasan terhadap teknik yang digunakan.
Teori	: Istilah dalam praxiologi untuk konsep yang berlaku umum di matematika untuk menjustifikasi beragam teknologi.
Transposisi	: Proses peralihan suatu objek menjadi objek yang lain.
Transposisi didaktik	: Proses transisi pengetahuan ilmiah yang dikembangkan ilmunan/matematikawan (<i>scholarly knowledge</i>) menjadi pengetahuan yang akan diajarkan (<i>knowledge to be taught</i>), pengetahuan yang diajarkan (<i>taught knowledge</i>) dan pengetahuan yang dipelajari (<i>learnt knowledge</i>).
Type of task	: Istilah dalam praxiologi untuk masalah yang akan diselesaikan