TRANSPOSISI DIDAKTIK DALAM KONSEP PERTIDAKSAMAAN: SEBUAH STUDI FENOMENOLOGI

DISERTASI

Diajukan untuk Memenuhi Sebagian dari Syarat Memperoleh Gelar Doktor Pendidikan Matematika

Oleh

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TRANSPOSISI DIDAKTIK DALAM KONSEP PERTIDAKSAMAN: SEBUAH STUDI FENOMENOLOGI

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ABSTRAK

Muhammad Daut Siagian (2023). Transposisi Didaktik dalam Konsep Pertidaksamaan: Sebuah Studi Fenomenologi.

Penelitian ini bertujuan menghasilkan pengetahuan yang terjastifikasi melalui penyelidikan bagaimana proses transposisi internal (dari *knowledge to be taught* ke *taught knowledge*) yang dialami oleh guru dalam mengajarkan konsep pertidaksamaan linear, serta mengeksplorasi pembentukan konsep pertidaksamaan pada diri siswa melalui eksplorasi pengalaman belajarnya dan melalui penyelidikan hambatan belajar yang dialaminya. Rancangan penelitian kualitatif dengan pendekatan fenomenologi dipilih untuk mencapai tujuan penelitian dengan melibatkan 22 siswa kelas 11. Hasil penelitan menunjukan bahwa sifat urutan pada $\mathbb{R}$ dan definisi "Misalkan a, $b \in \mathbb{R}$; Jika $a - b \in \mathbb{P}$, maka kita dapat menuliskannya sebagai $a > b$ (atau ekuivalen $b < a$); Jika $a - b \in \mathbb{P} \cup \{0\}$, maka kita dapat menuliskannya sebagai $a \geq b$ (atau ekuivalen $b \leq a$), merupakan dasar yang digunakan untuk menurunkan "aturan pertidaksamaan". Makna $a > b$ menurut *scholarly knowledge* yaitu, jika $a - b$ adalah positif, atau dengan kata lain selisih antara a dan b lebih dari nol. Sedangkan makna $a \geq b$, yaitu jika $a - b$ adalah positif dan gabungan 0. Namun, makna atau pengetahuan ini tidak diperkenalkan atau tidak sampai kepada siswa. Temuan penelitian juga menunjukkan bahwa sebagian besar siswa memahami pertidaksamaan sebagai membandingkan dua nilai. Juga, siswa cenderung memandang pertidaksamaan sebagai persamaan. Disamping itu, diperoleh tujuh ragam *concept image* siswa terkait dengan konsep pertidaksamaan, yaitu perkalian dan pembagian terhadap bilangan negatif tidak merubah tanda ketaksamaan; generalisasi konsep pertidaksamaan linear yang melibatkan nilai mutlak pada konsep pertidaksamaan linear satu variabel; tidak ada perbedaan makna notasi interval terbuka dan tertutup dalam mengekspresikan bentuk pertidaksamaan; pertidaksamaan sebagai alat untuk membandingkan besaran-besaran yang diketahui; pertidaksamaan sebagai ekspresi dan proses aljabar; sistem pertidaksamaan linear dua variabel sebagai teknik dalam menyelesaikan pertidaksamaan linear dua variabel; dan hanya ada satu nilai bilangan real dari variabel dalam pertidaksamaan yang membuat pertidaksamaan itu benar. Secara keseluruhan, berdasarkan eksplorasi pemahaman siswa terhadap konsep pertidaksamaan ditemukan adanya hambatan belajar (ontogenik, didaktik, dan epistemologis) yang dialami siswa.

Kata Kunci: Transposisi didaktik; pertidaksamaan; *concept image*; praksiologi; *learning obstacle*.

ABSTRACT

Muhammad Daut Siagian (2023). Didactic Transposition in the Concept of Inequality: A Phenomenological Study.

This study aims to explore teacher experience through investigating how the internal transposition process (from knowledge to be taught to taught knowledge) is carried out by teachers in teaching the concept of linear inequality, as well as exploring the meaning of the concept of inequality in students through exploring their learning experiences and investigating learning obstacles. The research design with a qualitative approach using the phenomenological method was chosen to achieve the research objectives by involving 22 students in grade 11. The research results show that the nature of the order in $\mathbb{R}$ and the definition of "Suppose a, $b \in \mathbb{R}$; if $a - b \in \mathbb{P}$, then we can write it as $a > b$ (or equivalently $b < a$); if $a - b \in \mathbb{P} \cup \{0\}$, then we can write it as $a \geq b$ (or equivalently $b \leq a$), which is the basis used to derive the "rule of inequality". According to scholarly knowledge, $a > b$ means that $a - b$ is positive, or that the difference between a and b is greater than zero. While the meaning of $a \geq b$ is if $a - b$ is positive and the combination is 0. However, this meaning or knowledge is not introduced or does not reach students. The research findings also show that most students understand inequality as a comparison of two values. Also, students tend to view inequality as equality. In addition, seven kinds of student concept images were obtained related to the concept of inequality, namely: multiplication and division of negative numbers do not change the sign of the inequality; generalization of the concept of linear inequalities involving absolute values in the concept of one-variable linear inequalities; there is no difference in the meaning of open and closed interval notations in expressing forms of inequality; inequalities as a tool to compare known quantities; inequalities as expressions and algebraic processes; a system of two-variable linear inequalities as a technique for solving two-variable linear inequalities. Overall, based on the exploration of students' understanding of the concept of inequality, it was found that there were learning obstacles (ontogenic, didactic, and epistemological) experienced by students.

Keywords: Didactic transposition; inequality; concept image; praxeology; learning obstacles.

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Muhammad Daut Siagian

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Abramson, J. (2015). *College algebra*. Houstan, Texas: Openstax.

Aebi, M. F., & Linde, A. (2015). The epistemological obstacles in comparative criminology: A special issue introduction. *European Journal of Criminology*, *12*(4), 381–385. https://doi.org/10.1177/1477370815595311

Akar, N., & Işıksal-Bostan, M. (2022). The didactic transposition of quadrilaterals: The case of 5th grade in Turkey. *International Journal of Mathematical Education in Science and Technology*, 1–22. https://doi.org/10.1080/0020739X.2021.2022228

Almog, N., & Ilany, B. S. (2012). Absolute value inequalities: High school students' solutions and misconceptions. *Educational Studies in Mathematics*, *81*(3), 347–364. https://doi.org/10.1007/s10649-012-9404-z

Araya, R., & Gormaz, R. (2021). Revealed Preferences of Fourth Graders When Requesting Face-to-Face Help While Doing Math Exercises Online. *Education Sciences*, *11*(8), 429. https://doi.org/10.3390/educsci11080429

Artigue, M. (1994). Didactical engineering as a framework for the conception of teaching products. Dalam R. Biehler, R. W. Scholz, R. Sträßer, & B. Winkelmann (Ed.), *Didactics of Mathematics as a Scientific Discipline* (hlm. 27–39). Netherlands: Dordrecht: Kluwer Academic Publishers.

Artigue, M., Haspekian, M., & Corblin-Lenfant, A. (2014). Introduction to the Theory of Didactical Situations (TDS). Dalam A. Bikner-Ahsbahs & S. Prediger (Ed.), *Networking of Theories as a Research Practice in Mathematics Education. Advances in Mathematics Education* (hlm. 47–65). Switzerland: Springer, Cham. https://doi.org/10.1007/978-3-319-05389-9_4

As'ari, A. R., Tohir, M., Valentino, E., Imron, Z., & Taufiq, I. (2017). *Matematika untuk SMP/MTs kelas VII semester 1*. Jakarta: Pusat Kurikulum dan Perbukuan, Balitbang, Kemendikbud.

Atalar, F. B., & Ergun, M. (2018). Evaluation of the knowledge of science teachers with didactic transposition theory. *Universal Journal of Educational Research*, *6*(1), 298–307. https://doi.org/10.13189/ujer.2018.060130

Bachelard, G. (1938). *La formation de l'esprit scientifique*. Paris: Librairie philosophique J. VRIN.

Bagni, G. T. (2005). Inequalities and equations: History and didactics. Dalam M. Bosch (Ed.), *Proceedings of CERME* (Vol. 4, hlm. 652–662).

Balacheff, N. (1990). Towards a problématique for research on mathematics teaching. *Journal for Research in Mathematics Education*, *21*(4), 258–272. https://doi.org/10.2307/749524

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools. *Educational Studies in Mathematics*, *59*(1–3), 235–268. https://doi.org/10.1007/s10649-005-5889-z

Bardini, C., & Pierce, R. (2015). Assumed mathematics knowledge: The challenge of symbols. *International Journal of Innovation in Science and Mathematics Education*, *23*(1), 1–9.

Barlovits, S., Jablonski, S., Lázaro, C., Ludwig, M., & Recio, T. (2021). Teaching from a distance—math lessons during COVID-19 in Germany and Spain. *Education Sciences*, *11*(8), 406. https://doi.org/10.3390/educsci11080406

Bartle, R. G., & Sherbert, D. R. (1983). *Introduction to real analysis*. New Jersey: John Wiley & Sons, Inc.

Baştürk, S. (2009). Student teachers' approaches to student's mistakes in the case of the absolute value concept. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *3*(1), 174–194.

Bazzini, L., & Tsamir, P. (2003). Connections between theory and research findings: The Case of inequalities. *European Research in Mathematics Education III*. Bellaria, Italy.

Bazzini, L., & Tsamir, P. (2004). Algebraic equations and inequalities: Issues for research and teaching. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2004*, *I*, 137–166.

Beckenbach, E., & Bellman, R. (1961). *An introduction to inequalities*. New York: Random House.

Bednarz, N., Kieran, C., & Lee, L. (1996). Approaches to algebra: Perspectives for research and teaching. Dalam N. Bednarz, C. Kieran, & L. Lee (Ed.), *Approaches to algebra*. Netherland: Kluwer Academic Publishers.

Bergsten, C., Jablonka, E., & Klisinska, A. (2010). A remark on didactic transposition theory. *Mathematics and mathematics education: Cultural and social dimensions: Proceedings of MADIF7, Stockholm, January 26-27, 2010*, 58–68.

Bicer, A., Capraro, R. M., & Capraro, M. M. (2014). Pre-service teachers' linear and quadratic inequalities understandings. *International Journal for Mathematics Teaching and Learning*, *15*(2).

Blanco, L. J., & Garrote, M. (2007a). Difficulties in learning inequalities in students of the first year of pre-university education in Spain. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(3), 221–229. https://doi.org/10.12973/ejmste/75401

Blanco, L. J., & Garrote, M. (2007b). Difficulties in learning inequalities in students of the first year of pre-university education in Spain. *Eurasia Journal of Mathematics, Science & Technology Education*, *3*(3), 221–229. https://doi.org/10.12973/ejmste/75401

Boesen, J., Lithner, J., & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics*, *75*(1), 89–105. https://doi.org/10.1007/s10649-010-9242-9

Bonoma, T. V. (1985). Case research in marketing: Opportunities, problems, and a process. *Journal of Marketing Research*, *22*(2), 199–208. https://doi.org/10.1177/002224378502200209

Booth, J. L., McGinn, K. M., Barbieri, C., & Young, L. K. (2017). Misconceptions and learning algebra. Dalam S. Stewart (Ed.), *And the Rest is Just Algebra*. Springer International Publishing Switzerland. https://doi.org/10.1007/978-3-319-45053-7_4

Booth, L. R. (1988). Children's difficulties in beginning algebra. Dalam A. F. Coxford & A. P. Shulte (Ed.), *The ideas of algebra K-12, 1988 yearbook of teachers of mathematics (NCTM)*. Reston, VA: NCTM.

Bosch, Marianna, & Chevallard, Y. (1999). The sensitivity of mathematical activity for ostensive. *Research in Maths Didactics*, *19*, 77–123.

Bosch, Marianna, Chevallard, Y., & Gascón, J. (2006). Science or magic? The use of models and theories in didactics of mathematics. *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education*, 1254–1263.

Bosch, Marianna, & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin*, (58), 51–65.

Botty, H. M. R. H., Yusof, H. J. H. M., Shahrill, M., & Mahadi, M. A. (2015). Exploring students' understanding on 'inequalities.' *Mediterranean Journal of Social Sciences*, *6*(5), 218–227. https://doi.org/10.5901/mjss.2015.v6n5s1p218

Brousseau, G. (2000). Education et didactique des mathématiques. *Educacion Matematica*, *12*(1), 5–39.

Brousseau, G. (2002). *Theory of didactical situations in mathematics*. New York: Kluwer Academic Publisher.

Brown, S. A. (2008). Exploring epistemological obstacles to the development of mathematics induction. *Proceedings of the 11th Conference for Research on Undergraduate Mathematics Education*, 1–19. San Diego, CA.

Cajori, F. (1993). *A history of mathematical notations*. Dover Publications.

Cavalcante, A., & Huang, H. (2022). Understanding Chinese students' success in the PISA financial literacy: A praxeological analysis of financial numeracy. *Asian Journal for Mathematics Education*, *1*(1), 66–94. https://doi.org/10.1177/27527263221091304

Chevallard, Y. (1986a). La transposition didactique: du savoir savant au savoir enseigné. *Revue française de pédagogie*, *76*(1), 89–91.

Chevallard, Y. (1986b). La transposition didactique: du savoir savant au savoir enseigné. *Revue française de pédagogie*, *76*(1), 89–91.

Chevallard, Y. (1989). On didatic transposition theory: Some introductory notes. *Paper presented at the International Symposium on Research and Development in Mathematics Education, Bratislava, Czechoslavak. Paru dans les Proceedings de ce symposium*, 51–62.

Chevallard, Y. (1992). A theoretical approach to curricula. *Journal für Mathematik-Didaktik: Zeitschrift der Gesellschaft für Didaktik der Mathematik (GDM)*, *13*(2), 215–230. https://doi.org/10.1007/BF03338779

Chevallard, Y. (1994). Ostensifs et non-ostensifs dans l'activité mathématique. *Intervention au Séminaire de l'Associazione Mathesis*, 190–200.

Chevallard, Y. (2006). Step towards a new epistemology in mathematics education. Dalam Marianna Bosch (Ed.), *Proceedings of the IV Congress of the European Society for Research in Mathematics Education* (hlm. 21–30). La Pensée Sauvage.

Chevallard, Y. (2007). Readjusting didactics to a changing epistemiology. *European Educational Research Journal*, *6*(2), 131–134. https://doi.org/10.2304/eerj.2007.6.2.131

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114.

Chevallard, Y., & Bosch, M. (2014). Didactic transposition in mathematics education. Dalam S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (hlm. 170–174). London: Springer Dordrecht Heidelberg New York London. https://doi.org/10.1145/1869746.1869759

Chevallard, Y., & Joshua, M. A. (1991). *La transposition didactique. Du savoir savant au savoir enseigné (nouv. éd. revue et augmentée)*. Grenoble: La Pensée Sauvage.

Chevallard, Y., & Sensevy, G. (2014). Anthropological approaches in mathematics education, French perspectives. Dalam S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (hlm. 38–43). Dordrecht: Springer.

Chirinda, B., Ndlovu, M., & Spangenberg, E. (2021). Teaching mathematics during the COVID-19 lockdown in a context of historical disadvantage. *Education Sciences*, *11*(4), 177. https://doi.org/10.3390/educsci11040177

Çiltaş, A., & Tatar, E. (2011). Diagnosing learning difficulties related to the equation and inequality that contain terms with absolute value. *International Online Journal of Educational Sciences*, *3*(2), 461–473.

Clément, P. (2006). Didactic transposition and the KVP model: Conceptions as interactions between scientific knowledge, values and social practices. *Paper presented at the Proceedings of the ESERA Summer School, Universidade do Minho, Braga, Portugal, 15-22 July*, 9–18.

Cloud, M. J., & Drachman, B. C. (1998). *Inequalities: With applications to engineering*. New York: Springer-Verlag.

Cloud, M. J., Drachman, B. C., & Lebedev, L. P. (2014). Inequalities: With applications to engineering. Dalam *Inequalities: With Applications to Engineering: Second Edition* (Second Edition). New York: Springer. https://doi.org/10.1007/978-3-319-05311-0

Coffey, A., & Atkinson, P. (1996). *Making sense of qualitative data: Complementary research strategies*. Thousand Oaks, CA: Sage.

Cooper, B., & Harries, T. (2002). Children's responses to contrasting "realistic" mathematics problems: Just how realistic are children ready to be? *Educational Studies in Mathematics*, *49*(1), 1–23. https://doi.org/10.1023/a:1016013332659

Cornu, B. (1981). Apprentissage de la notion de limite: Modèles spontanés et modèles propres. *Proccedings PME (Psychology of Mathematics Education)*, 322–326. Grenoble, France.

Cornu, B. (1991). Limits. Dalam D. Tall (Ed.), *Advanced Mathematical Thinking* (Vol. 11, hlm. 153–166). Dordrecht: Springer Netherlands. https://doi.org/10.1007/0-306-47203-1_10

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA: Sage.

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (Fourth Edition). Boston, MA: Pearson Education, Inc.

Davies, C., & Peck, W. G. (1855). *Mathematical dictionary and encyclopedia of mathematical science*. New York: A. S. Barnes & Co.

Davis, N. L., Gough, M., & Taylor, L. L. (2019). Online teaching: Advantages, obstacles and tools for getting it right. *Journal of Teaching in Travel & Tourism*, *19*(3), 256–263. https://doi.org/10.1080/15313220.2019.1612313

de Grefte, J. (2021). Knowledge as justified true belief. *Erkenntnis*. https://doi.org/10.1007/s10670-020-00365-7

de Sousa, R. T., Alves, F. R. V., & Souza, M. J. A. (2022). An examination about the didactic transposition in the post-remote teaching scenario: Obstacles and possibilities. *European Journal of Education Studies*, *9*(8), 154–175. https://doi.org/10.46827/ejes.v9i8.4414

Demir, E., & Demir, C. G. (2021). Investigation of parents' opinions about distance education during the covid-19 pandemic. *Turkish Online Journal of Distance Education-TOJDE*, *22*(2), 42–57.

Denbel, D. G. (2013). Students Difficulties of Solving Inequalities in Calculus. *American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)*, *5*(1), 57–62.

Djelil, F., & Sanchez, E. (2023). Game design and didactic transposition of knowledge. The case of progo, a game dedicated to learning object-oriented programming. *Education and Information Technologies*, *28*(1), 283–302. https://doi.org/10.1007/s10639-022-11158-6

Do, T. H., & Nguyen, V. T. T. (2020). The structure of didactic transposition capability-Analysis of an example of didactic transposition of physical knowledge in the training of pedagogical students. *Vietnam Journal of Education*, *4*(1), 44–52.

Dobbs, D. E., & Peterson, J. C. (1991). The sign-chart method for solving inequalities. *The Mathematics Teacher*, *84*(8), 657–664.

Dong, A., Jong, M. S.-Y., & King, R. B. (2020). How does prior knowledge influence learning engagement? The mediating roles of cognitive load and help-seeking. *Frontiers in Psychology*, *11*, 591203. https://doi.org/10.3389/fpsyg.2020.591203

Dreyfus, T., & Eisenberg, T. (1985). A graphical approach to solving inequalities. *School Science and Mathematics*, *85*(8), 651–662. https://doi.org/10.1111/j.1949-8594.1985.tb09678.x

Dutant, J. (2015). The legend of the justified true belief analysis. *Philosophical Perspectives*, *29*(1), 95–145. https://doi.org/10.1111/phpe.12061

EdSource. (2009). Why it is important to learn algebra. www.edsource.org

Edwards, B. S., & Ward, M. B. (2004). Surprises from mathematics education research: Student (mis)use of mathematical definitions. *The American Mathematical Monthly*, *111*(5), 411–424. https://doi.org/10.2307/4145268

Elbaz, F. (1983). *Teacher thinking: A study of practical knowledge*. London: Croom Helm.

Elia, I., Özel, S., Gagatsis, A., Panaoura, A., & Özel, Z. E. Y. (2016). Students' mathematical work on absolute value: focusing on conceptions, errors and obstacles. *ZDM - Mathematics Education*, *48*(6), 895–907. https://doi.org/10.1007/s11858-016-0780-1

Elias, H. R., Ribeiro, A. J., & Savioli, A. M. P. das D. (2020). Epistemological matrix of rational number: A look at the different meanings of rational numbers. *International Journal of Science and Mathematics Education*, *18*(2), 357–376. https://doi.org/10.1007/s10763-019-09965-4

El-Khateeb, M. M. A. (2016). Errors analysis of solving linear inequalities among the preparatory year students at king saud university. *Journal of Education and Practice*, *7*(12), 124–133.

Ellerton, N. F., & Clements, M. A. (Ken). (2011). Prospective middle-school mathematics teachers' knowledge of equations and inequalities. Dalam J. Cai & E. J. Knuth (Ed.), *Early Algebraization, Advances in Mathematics Education*. Verlag Berlin Heidelberg: Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-17735-4

El-Shara', I. A., & Al-Abed, A. S. (2010). Errors analysis of solving inequalities among mathematics majors at the University of Jordan. *Jordan Journal of Educational Science*, *6*(2), 93–108.

Erbaş, K., Çetinkaya, A., & Yaşar, E. (2009). Student difficulties and misconceptions in solving simple linear equations. *Education and Science*, *34*(152), 44–59.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. Dalam Douglas A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. New York: Macmillan.

Fischbein, E., & Barash, A. (1993). Algorithmic models and their misuse in solving algebraic problems. *Proceedings of the 17th International Conference on the Psychology of Mathematics Education* , 162–172. Tsukuba, Japan.

Florensa, I., Bosch, M., & Gascón, J. (2015). The epistemological dimension in didactics: Two problematic issues. *CERME 9 - Ninth Congress of the European Society for Research in MathematicsEducation*, 2635–2641. Prague, Czech Republic: Charles University, Faculty of Education.

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal*, *26*(3), 403–421. https://doi.org/10.3102/00028312026003403

Frempong, A. A. (2012). *Final exam review: Intermediate algebra*. New York: Finalexamsreview.com.

Fusch, P., Fusch, G., & Ness, L. (2017). How to conduct a mini-ethnographic case study: A guide for novice researchers. *The Qualitative Report*, *22*(3), 923–941. https://doi.org/10.46743/2160-3715/2017.2580

Gellert, U., Barbé, J., & Espinoza, L. (2013). Towards a local integration of theories: Codes and praxeologies in the case of computer-based instruction. *Educational Studies in Mathematics*, *82*(2), 303–321. https://doi.org/10.1007/s10649-012-9427-5

Gellert, W., Kustner, H., & Hellwich, M. (1975). *Mathematics at a glance: A compendium*. Leipzig: VEB Bibliographisches Institut.

Gök, M., Erdoğan, A., & Erdoğan, E. Ö. (2019). Transpositions of function concept in mathematics curricula and textbooks from the historical development perspective. *International Journal of Instruction*, *12*(1), 1189–1206. https://doi.org/10.29333/iji.2019.12176a

González-Martín, A. S. (2015). The use of textbooks by pre-university teachers: Anexample with infinite series of real numbers. Dalam K. Krainer & N. Vondrová (Ed.), *Proceedings of the ninth congress of the European society for research in math- ematics education* (hlm. 2124–2130). Prague: Charles University in Prague, Faculty of Education and ERME.

Grbich, C. (2007). *Qualitative data analysis: An introduction*. London: Sage Publication Inc.

Groenewald, T. (2004). A phenomenological research design illustrated. *International Journal of Qualitative Methods*, *3*(1), 42–55. https://doi.org/10.1177/160940690400300104

Gustafson, R. D., & Frisk, P. D. (2008). *Intermediate algebra*. United States of America: Thomson Learning, Inc.

Hall, R. D. G. (2002). An analysis of errors made in the solution of simple linear equations. *Philosophy of Mathematics Education Journal*, *15*.

Halmaghi, E. F. (2011). *Undergraduate students' conceptions of inequalities* (Dissertation). Simon Fraser University, Canada.

Harel, G. (2008). What is Mathematics? A Pedagogical Answer to a Philosophical Question. Dalam B. Gold & R. A. Simons (Ed.), *Proof and Other Dilemmas: Mathematics and Philosophy*. United States of America: The Mathematical Association of America, Inc.

Hausberger, T. (2017). The (homo)morphism concept: Didactic transposition, meta-discourse and thematisation. *International Journal of Research in Undergraduate Mathematics Education*, *3*(3), 417–443. https://doi.org/10.1007/s40753-017-0052-7

Hayashi, P., Abib, G., & Hoppen, N. (2019). Validity in qualitative research: A processual approach. *The Qualitative Report*, *24*(1), 98–112. https://doi.org/10.46743/2160-3715/2019.3443

Herstein, I. N. (1964). *Topics in algebra*. New York: John Wiley & Sons, Inc.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. Dalam J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (hlm. 1–27). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Inc., Publishers.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406. https://doi.org/10.3102/00028312042002371

Holloway, I. (1997). *Basic concepts for qualitative research*. Oxford: Blackwell Science.

Hwang, S., & Cho, E. (2021). Exploring latent topics and research trends in mathematics teachers' knowledge using topic modeling: A systematic review. *Mathematics*, *9*(22), 2956. https://doi.org/10.3390/math9222956

Hycner, R. H. (1985). Some guidelines for the phenomenological analysis of interview data. *Human Studies*, *8*(3), 279–303.

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education*, *38*(3), 258–288.

Jäder, J., Lithner, J., & Sidenvall, J. (2019). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*, *51*(7), 1–17. https://doi.org/10.1080/0020739X.2019.1656826

Jaworski, B., & Gellert, U. (2003). Educating new mathematics teachers: Integrating theory and practice, and the roles of practising teachers. Dalam A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Ed.), *Second International Handbook of Mathematics Education* (hlm. 829–875).

Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-010-0273-8_27

Jick, T. D. (1979). Mixing qualitative and quantitative methods: Triangulation in action. *Administrative Science Quarterly*, *24*(4), 602–611. https://doi.org/10.2307/2392366

Johansson, M. (2006). *Teaching mathematics with textbooks: A classroom and curricular perspective* (Dissertation). Luleå University of Technology, Sweden.

Johnson, A. (1994). *Classic math: History topics for the classroom*. Palo Alto, Sa: Dale Seymour Publications.

Jupri, A., Usdiyana, D., & Gozali, S. M. (2022). Pre-Service teachers' strategies in solving absolute value equations and inequalities. *Education Sciences*, *12*(11), 743. https://doi.org/10.3390/educsci12110743

Kalogeropoulos, P., Roche, A., Russo, J., Vats, S., & Russo, T. (2021). Learning mathematics from home during COVID-19: Insights from two inquiry-focussed primary schools. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(5), em1957. https://doi.org/10.29333/ejmste/10830

Kang, W. A. N., & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. *For the Learning of Mathematics*, *12*(1), 2–7.

Kansanen, P. (2003). Studying-the realistic bridge between instruction and learning. An attempt to a conceptual whole of the teaching-studying-learning process. *Educational Studies*, *29*(2–3), 221–232. https://doi.org/10.1080/03055690303279

Kansanen, P., Tirri, K., Meri, M., Krokfors, L., Husu, J., & Jyrhämä, R. (2000). *Teachers' pedagogical thinking. Theoretical landscapes, practical challenges*. New York: Peter Lang Publishing, Inc. https://doi.org/10.3726/978-1-4539-1514-1

Kaplan, M. (1985). It's not what you know that counts. *The Journal of Philosophy*, *82*(7), 350–363. https://doi.org/10.2307/2026524

Kaput, James J. (2008). What is algebra? What is algebraic reasoning? Dalam J. J. Kaput, D. W. Carrahe, & M. L. Blanton (Ed.), *Algebra In The Early Grades*. New York: Lawrence Erlbaum Associates.

Katz, V. J. (2007). *Algebra: Gateway to a technological future*. Columbia: University of the District of Columbia.

Kazarinoff, N. D. (1961). *Geometric inequalities*. New York: Random House.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, *12*(3), 317–326. https://doi.org/10.1007/BF00311062

Kieran, C. (1992). The learning and teaching of school algebra. Dalam D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Kieran, C. (2007). Research on the learning and teaching of algebra. Dalam A. Gutiérrez & P. Boero (Ed.), *Handbook of Research on the Psychology of Mathematics Education* (hlm. 256–258). Rotterdam: BRILL. https://doi.org/10.1163/9789087901509_015

Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). *Early algebra: Research into its nature, its learning, its teaching*. Switzerland: Springer International Publishing.

King, N. (1994). The qualitative research interview. Dalam C. Cassell & G. Symon (Ed.), *Qualitative methods in organisational research: A practical guide*. London: Sage.

Knuth, E. J. (2000). Student understanding of the cartesian connection: An exploratory study. *Journal for Research in Mathematics Education*, *31*(4), 500–508. https://doi.org/10.2307/749655

Kriegler, S. (2007). Just what is algebraic thinking? *Introduction to algebra:Teacher Handbook*, 1–11.

Lacina, J., & Block, C. C. (2011). What matters most in distinguished literacy teacher education programs? *Journal of Literacy Research*, *43*(4), 319–351. https://doi.org/10.1177/1086296X11422033

Langdridge, D. (2007). *Phenomenological psychology: Theory, research and methods*. London: Pearson.

Larson, R., Hostetler, R. P., & Edwards, B. H. (2005). *College algebra: A graphing approach*. New York: Houghton Mifflin Company.

Lehrer, K., & Paxson, T. (1969). Knowledge: Undefeated justified true belief. *The Journal of Philosophy*, *66*(8), 225–237. https://doi.org/10.2307/2024435

Lim, K. H. (2006). *Students' mental acts of anticipating in solving problems involving algebraic inequalities and equations* (Dissertation, University of California). University of California, San Diego.

Linchevski, L., & Sfard, A. (1991). Rules without reasons as processes without objects – The case of equations and inequalities. Dalam F. Furinghetti (Ed.), *Procedings of the 15th Conference of the International Group for the Psychology of Mathematics Education(PME)* (Vol. 2, hlm. 317–324). Asisi, Italy: Psychology of Mathematics Education.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage Publication.

Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics*, *41*(2), 165–190. https://doi.org/10.1023/A:1003956417456

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics*, *52*(1), 29–55. https://doi.org/10.1023/A:1023683716659

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2

Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM Mathematics Education*, *49*(6), 937–949. https://doi.org/10.1007/s11858-017-0867-3

Love, E., & Pimm, D. (1996). "This is so": A text on texts. Dalam A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Ed.), *nternational Handbook of Mathematics Education. Kluwer International Handbooks of Education* (Vol. 4, hlm. 371–409). Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-009-1465-0_11

Lundberg, A. L. V., & Kilhamn, C. (2018). Transposition of knowledge: Encountering proportionality in an algebra task. *International Journal of*

Science and Mathematics Education, *16*(3), 559–579. https://doi.org/10.1007/s10763-016-9781-3

Macgregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics*, *33*(1), 1–19. https://doi.org/10.1023/A:1002970913563

Maknun, C. L., Rosjanuardi, R., & Jupri, A. (2022). Epistemological obstacle in learning trigonometry. *Mathematics Teaching Research Journal*, *14*(2), 5–25.

Makonye, J., & Shingirayi, M. (2014). The obstacles faced by the learners in the learning of quadratic inequalities. *Mediterranean Journal of Social Sciences*, *5*(27), 716–725. https://doi.org/10.5901/mjss.2014.v5n27p716

Mangiante-Orsola, C., Perrin-Glorian, M.-J., & Strømskag, H. (2018). Theory of didactical situations as a tool to understand and develop mathematics teaching practices. *Annales de didactique et de sciences cognitives*, (Special issue), 145–174. https://doi.org/10.4000/adsc.334

Mclaurin, S. C. (1985). A unified way to teach the solution of inequalities. *The Mathematics Teacher*, *78*(2), 91–95.

Mello, L. A. de. (2017). A propose of rules defining as a didactic transposition should occur or be achieved - The generalized didactic transposition theory. https://doi.org/10.31219/osf.io/uzfhb

Mello, L. A. De. (2019). *A Propose of Rules Defining as a Didactic Transposition Should Occur or be Achieved - The Generalized Didactic Transposition Theory*. (1982).

Mihalca, L., Salden, R. J. C. M., Corbalan, G., Paas, F., & Miclea, M. (2011). Effectiveness of cognitive-load based adaptive instruction in genetics education. *Computers in Human Behavior*, *27*(1), 82–88. https://doi.org/10.1016/j.chb.2010.05.027

Miyakawa, T., & Winsløw, C. (2013). Developing mathematics teacher knowledge: The paradidactic infrastructure of "open lesson" in Japan. *Journal of Mathematics Teacher Education*, *16*(3), 185–209. https://doi.org/10.1007/s10857-013-9236-5

Modestou, M., & Gagatsis, A. (2007). Students' improper proportional reasoning: A result of the epistemological obstacle of "linearity." *Educational Psychology: An International Journal of Experimental Educational Psychology*, *27*(1), 75–92. https://doi.org/10.1080/01443410601061462

Mok, I. A. C., & Clarke, D. J. (2015). The contemporary importance of triangulation in a post-positivist world: Examples from the learner's perspective study. Dalam A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Ed.), *Approaches to Qualitative Research in Mathematics Education. Advances in Mathematics Education* (hlm. 403–425). Dordrecht: Springer. https://doi.org/10.1007/978-94-017-9181-6_15

Mokh, R. A., Othman, A., & Shahbari, J. A. (2019). Mistakes made by students with logical connectives when solving equations and inequalities, and how teachers assess these mistakes. *International Journal of Research in Education and Science (IJRES)*, *5*(2), 421–428.

Moon, K. (2019). Graphs of two variable inequalities: Alternate approaches to the solution test. *The Mathematics Enthusiast*, *16*(1–3), 107–126. https://doi.org/10.54870/1551-3440.1452

Moru, E. K. (2007). Talking with the literature on epistemological obstacles. *For the Learning of Mathematics*, *27*(3), 34–37.

Moustakas, C. (1994). *Phenomenological research methods*. Thousand Oaks, CA: Sage.

Murtiyasa, B. (2016). Isu-isu kunci dan tren penelitian pendidikan matematika. *Prosiding Konferensi Nasional Penelitian Matematika dan Pembelajarannya (KNPMP) I 2016*, 1–10. Surakarta: Universitas Muhammadiyah Surakarta.

Mutodi, P., & Mosimege, M. (2021). Learning mathematical symbolization: Conceptual challenges and instructional strategies in secondary schools. *Bolema - Mathematics Education Bulletin*, *35*(70), 1180–1199. https://doi.org/10.1590/1980-4415v35n70a29

Myhill, D., & Brackley, M. (2004). Making connections: Teachers' use of children's prior knowledge in whole class discourse. *British Journal of Educational Studies*, *52*(3), 263–275. https://doi.org/10.1111/j.1467-8527.2004.00267.x

Naidoo, I., & Huntley, B. (2010). Procepts and proceptual solutions in undergraduate mathematics . *Pythagoras*, (72), 36–49. https://doi.org/10.4102/pythagoras.v0i72.56

National Mathematics Advisory Panel. (2008). *Fondation for success: The final report of the national mathematics advisory panel*. Washington, DC: U.S. Department of Education. https://doi.org/10.3102/0013189X08329195

National Research Council. (2001). *Improving Mathematics Education: Resources for Decision Making* (S. Leinwand & G. Burril, Ed.). Washington, DC: The National Academies Press. https://doi.org/10.17226/10268

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

NCTM. (2000). *Principles standards and for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Nditafon, G. N., & Noumi, E. (2018). Didactic transposition for inferential and analogical thinking, reasoning and transfer of school knowledge for societal context-of-use. *Open Access Library Journal*, *5*(e4785), 1–29. https://doi.org/10.4236/oalib.1104785

Nebesniak, A. L. (2012). Effective instruction: A mathematics coach's perspective. *The Mathematics Teacher*, *106*(5), 354–358. https://doi.org/10.5951/mathteacher.106.5.0354

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. Dalam A. Gagatsis & S. Papastavridis (Ed.), *roceedings of the 3rd Mediterranean Confer-ence on mathematics education; 2003 Jan 3–5; Athens* (hlm. 116–124). Athens, Greece: Hellenic Mathematical Society.

Nyikahadzoyi, M. R., Mapuwei, T., & Chinyoka, M. (2013). Some cognitive obstacles faced by "A" level mathematics students in understanding inequalities: A case study of bindura urban high schools. *International Journal of Academic Research in Progressive Education and Development*, *2*(2), 206–221.

Olson, J. D., McAllister, C., Grinnell, L., Gehrke Walters, K., & Appunn, F. (2016). Applying constant comparative method with multiple investigators and inter-

coder reliability. *The Qualitative Report*, *21*(1), 26–42. https://doi.org/10.46743/2160-3715/2016.2447

Osana, H. P., & Royea, D. A. (2011). Obstacles and challenges in preservice teachers' explorations with fractions: A view from a small-scale intervention study. *Journal of Mathematical Behavior*, *30*(4), 333–352. https://doi.org/10.1016/j.jmathb.2011.07.001

Østergaard, K. (2017). Theory and practice in mathematics teacher education. Dalam G. Cirade, M. Artaud, M. Bosch, J.-P. Bourgade, Y. Chevallard, C. Ladage, & T. Sierra (Ed.), *Évolutions contemporaines du rapport aux mathématiques et aux autres savoirs à l'école et dans la société* (hlm. 899–918). Toulouse: IVe congrès international sur la TAD.

Otero, M. R., & Llanos, C. (2021). The gap between studying a generating question and planning lessons based on it. Dalam *Trends in Mathematics* (Vol. 13, hlm. 67–76). Springer Science and Business Media Deutschland GmbH. https://doi.org/10.1007/978-3-030-76413-5_8

Oudina, A. S. (2020). La transposition didactiquedans l'enseignementdes langues étrangères. *Revue Langues, cultures et sociétés*, *6*(2), 132–138.

Palupi, E. L. W., Sumarto, S. N., & Purbaningrum, M. (2022). Senior high school students' understanding of mathematical inequality. *Jurnal Elemen*, *8*(1), 201–215. https://doi.org/10.29408/jel.v8i1.4537

Pansell, A., & Boistrup, L. B. (2018). Mathematics teachers' teaching practices in relation to textbooks: Exploring praxeologies. *The Mathematics Enthusiast*, *15*(3), 541–562.

Park, K. (2013). A study on didactic transposition of mathematics textbooks and lessons in Korea and the U.S. *Journal of the Korean School Mathematics Society*, *16*(2), 459–478.

Patton, M. Q. (1999). Enhancing the quality and credibility of qualitative analysis. *Health services research*, *34*(5), 1189–1208.

Pessanha, M., & Pietrocola, M. (2017). Particle accelerators and didactic obstacles. Dalam M. Pietrocola & I. Gurgel (Ed.), *Crossing the Border of the Traditional Science Curriculum: Innovative Teaching and Learning in Basic Science Education* (hlm. 45–59). Rotterdam: SensePublishers. https://doi.org/10.1007/978-94-6351-041-7_3

Piburn, M., & Sawada, D. (2000). *Reformed teaching observation protocol (RTOP): Reference manual*. Arizona.

Piez, C. M., & Voxman, M. H. (1997). Multiple representations—Using different perspectives to form a clearer picture. *The Mathematics Teacher*, *90*(2), 164–166.

Pinto, Marcia, & Tall, D. (1996). Student teachers' conceptions of the rational numbers. *Proceedings of the 20th Conference of the Internal Group for the Psychology of Mathematics Education (PME)*, *4*, 139–146.

Pinto, Márcia, & Tall, D. (2002). Building formal mathematics on visual mmagery: A case study and a theory. *For the Learning Mathematics*, *22*(1), 2–10. https://doi.org/10.2307/40248377

Postelnicu, V., & Coatu, S. (1980a). *Mica enciclopedie matematica (Small mathematical encyclopedia)*. Romania: Editura Technica Bucuresti.

Postelnicu, V., & Coatu, S. (1980b). *Mica enciclopedie matematica (Small mathematical encyclopedia)*. Romania: Editura Technica Bucuresti.

Postelnicu, Valentina. (2017). Didactic transposition in school algebra: The case of writing equations of parallel and perpendicular lines. *Tenth Congress of the European Society for Research in Mathematics Education CERME*, (10 Feb 2017), 480–487. Dublin, Ireland.

Pratiwi, Y., & Rosjanuardi, R. (2020). Error Analysis in Solving the Rational and Irrational Inequalities. *Proceedings of the Proceedings of the 7th Mathematics, Science, and Computer Science Education International Seminar, MSCEIS 2019, 12 October 2019, Bandung, West Java, Indonesia*. EAI. https://doi.org/10.4108/eai.12-10-2019.2296414

Prediger, S., Barzel, B., Hußmann, S., & Leuders, T. (2021). Towards a research base for textbooks as teacher support: The case of engaging students in active knowledge organization in the KOSIMA project. *ZDM – Mathematics Education*, *53*(6), 1233–1248. https://doi.org/10.1007/s11858-021-01245-2

Putra, Z. H. (2019). Praxeological change and the density of rational numbers: The case of pre-service teachers in Denmark and Indonesia. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(5), em1711. https://doi.org/10.29333/ejmste/105867

Putra, Z. H. (2020). Didactic transposition of rational numbers: A case from a textbook analysis and prospective elementary teachers' mathematical and didactic knowledge. *Journal of Elementary Education*, *13*(4), 365–394. https://doi.org/10.18690/rei.13.4.365-394.2020

Putra, Z. H., & Winsløw, C. (2018). Teachers' collective knowledge: The case of equivalent fractions. *Journal of Physics: Conference Series*, *1088*. Institute of Physics Publishing. https://doi.org/10.1088/1742-6596/1088/1/012003

Quessada, M. P., & Clément, P. (2007). An epistemological approach to French syllabi on human origins during the 19th and 20th centuries. *Science and Education*, *16*(9–10), 991–1006. https://doi.org/10.1007/s11191-006-9051-9

Radatz, H. (1980). Students' errors in the mathematical learning process: A survey. *For the Learning of Mathematics*, *1*(1), 16–20.

Radford, L., Bussi, M. G. B., Bekken, O., Boero, P., Dorier, J.-L., Katz, V., … Vasco, C. (2000). Historical formation and student understanding of mathematics. Dalam J. Fauvel & J. van Maanen (Ed.), *History in mathematics education: The ICMI study* (hlm. 143–170). Dordrecht: Kluwer.

Ramdhani, S., Suryadi, D., & Prabawanto, S. (2021). Hambatan belajar matematika di pondok pesantren. *Jurnal Analisa*, *7*(1), 46–55. https://doi.org/10.15575/ja.v7i1.10106

Rasmussen, K. (2016). Lesson study in prospective mathematics teacher education: didactic and paradidactic technology in the post-lesson reflection. *Journal of Mathematics Teacher Education*, *19*(4), 301–324. https://doi.org/10.1007/s10857-015-9299-6

Rips, L. J., Bloomfield, A., & Asmuth, J. (2008). From numerical concepts to concepts of number. *Behavioral and Brain Sciences*, *31*(6), 623–642. https://doi.org/10.1017/S0140525X08005566

Rosyidi, A. H., & Kohar, A. W. (2018). Student teachers' proof schemes on proof tasks involving inequality: Deductive or inductive? *Journal of Physics: Conference Series*, *947*, 012028. https://doi.org/10.1088/1742-6596/947/1/012028

Rowntree, R. V. (2007). Students' understandings and misconceptions of algebraic inequalities. *Research in Brief*, *109*(6), 311–312.

Rubenstein, R. N., & Thompson, D. R. (2001). Learning mathematical symbolism: Challenges and instructional strategies. *Mathematics Teacher*, *94*(4), 265–271. https://doi.org/10.5951/mt.94.4.0265

Rudi, R., Suryadi, D., & Rosjanuardi, R. (2022). Didactical transposition within reflective practice of an Indonesian mathematics teacher community: A case in proving the pythagorean theorem topic. *Southeast Asian Mathematics Education Journal*, *12*(1), 65–80. https://doi.org/10.46517/seamej.v12i1.132

Sackur, K. (2004). Problems related to the use of graphs in solving inequalities. Dalam M. J. Hoines & A. B. Fuglestad (Ed.), *Proceedings of the 28th Conference of the International Group of Psychology of Mathematics Education* (hlm. 148–152). Bergen, Norway: PME.

Sadala, M. L. A., & Adorno, R. dec. F. (2001). Phenomenology as a method to investigate the experiences lived: A perspective from Husserl and Merleau-Ponty"s thought. *Journal of Advanced Nursing*, *37*(3), 282–293.

Sadler, P. M., Sonnert, G., Coyle, H. P., Cook-Smith, N., & Miller, J. L. (2013). The influence of teachers' knowledge on student learning in middle school physical science classrooms. *American Educational Research Journal*, *50*(5), 1020–1049. https://doi.org/10.3102/0002831213477680

Salihu, L., & Räsänen, P. (2018). Mathematics skills of Kosovar primary school children: A special view on children with mathematical learning difficulties. *lnternational Electronic Journal of Elementary Education*, *10*(4), 421–430. https://doi.org/10.26822/iejee.2018438132

Sari, D. N., Usodo, B., & Pramudya, I. (2019). Analysis of concept understanding student in class X inequalities material. *Journal of Physics: Conference Series*, *1306*(1), 012029. https://doi.org/10.1088/1742-6596/1306/1/012029

Scheiner, T., Godino, J. D., Montes, M. A., Pino-Fan, L. R., & Climent, N. (2022). On metaphors in thinking about preparing mathematics for teaching. *Educational Studies in Mathematics*, *111*(2), 253–270. https://doi.org/10.1007/s10649-022-10154-4

Schneider, M. (2014). Epistemological obstacles in mathematics education. Dalam *Encyclopedia of Mathematics Education* (hlm. 214–217). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8

Schoenfeld, A. H., & Arcavi, A. (1988). On the meaning of variable. *Mathematics Teacher*, *81*(6), 420–427. https://doi.org/10.4013/fsu.2019.202.04

Schröder, E., Gredebäck, G., Forssman, L., & Lindskog, M. (2022). Predicting children's emerging understanding of numbers. *Developmental Science*, *25*(3), e13207. https://doi.org/10.1111/desc.13207

Sedrakyan, H., & Sedrakyan, N. (2018). *Algebraic inequalities*. Switzerland: Springer Nature .

Sequeira, A. H. (2012). Introduction to concepts of teaching and learning. *SSRN Electronic Journal*, 1–6. https://doi.org/10.2139/ssrn.2150166

Sevimli, E. (2022). Evaluation of the didactic transposition process in teaching integral: Face-to-face versus online education. *International Journal for Technology in Mathematics Education*, *29*(1), 37–48.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*(1), 1–36. https://doi.org/10.1007/BF00302715

Sidik, G. S., Suryadi, D., & Turmudi, T. (2021). Learning obstacle on addition and subtraction of primary school students: Analysis of algebraic thinking. *Education Research International*, *2021*, 5935179. https://doi.org/10.1155/2021/5935179

Sierpinska, A. (1987). Humanities students and epistemological obstacles related to limits. *Educational Studies in Mathematics*, *18*, 371–397. https://doi.org/10.1007/BF00240986

Sinaga, B., Sinambela, P. N. J. M., Sitanggang, A. K., Hutapea, T. A., Manulang, S., Sinaga, L. P., & Simanjorang, M. (2017). *Matematika untuk SMA/MA/SMK/MAK kelas X semester 1*. Jakarta: Pusat Kurikulum dan Perbukuan, Balitbang, Kemendikbud.

Sinaga, B., Sinambela, P. N. J. M., Sitanggang, A. K., Hutapea, T. A., Sinaga, L. P., Manullang, S., … Bayuzetra, Y. T. (2014). *Matematika untuk SMA/MA/SMK/MAK kelas X semester 1*. Jakarta: Pusat Kurikulum dan Perbukuan, Balitbang, Kemdikbud.

Sitanggang, R. A., & Araiku, J. (2022). Student mathematical modeling ability with blended learning approach on two linear inequality materials. *Advances in Social Science, Education and Humanities Research*, *656*, 177–182.

Smith, C., Julie, C., & Gierdien, F. (2022). Teaching mathematics in the fourth industrial revolution: Instances of instrumental orchestration when a teacher integrates GeoGebra in solving linear inequalities. Dalam *Mathematics Education in Africa* (hlm. 195–209). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-13927-7_12

Sriraman, B., & English, L. (2010). Surveying theories and philosophies of mathematics education. Dalam B. Sriraman & L. English (Ed.), *Theories of Mathematics Education. Advances in Mathematics Education* (hlm. 7–32). Berlin Heidelberg: Springer-Verlag. https://doi.org/10.1007/978-3-642-00742-2_2

Straehler-Pohl, H., & Gellert, U. (2013). Towards a Bernsteinian language of description for mathematics classroom discourse. *British Journal of Sociology of Education*, *34*(3), 313–332. https://doi.org/10.1080/01425692.2012.714250

Suhaedi, D. (2013). Peningkatan kemampuan komunikasi matematis, berpikir aljabar, dan disposisi matematis siswa smp melalui pendekatan pendidikan matematika realistik. Universitas Pendidikan Indonesia.

Suryadi, D. (2016). Refleksi kritis tradisi pendidikan matematika dan sebuah gagasan alternatif. Dalam F. Abdulhamid (Ed.), *Pemikiran Guru Besar tentang Pendidikan Abad 21*. Bandung: UPI Press.

Suryadi, D. (2019a). *Landasan filosofis penelitian desain didaktis (DDR)*. Bandung: Gapura Press.

Suryadi, D. (2019b). Pengetahuan transposisi sebagai konektor pendidikan akademik dan pendidikan profesi guru (PPG) matematika. Dalam *Disampaikan di Lingkungan Departemen Pendidikan Matematika Kamis, 15 Agustus 2019* (hlm. 1–23).

Symonds, P. M. (1922). The psychology of errors in algebra. *The Mathematics Teacher*, *15*(2), 93–104.

Tabach, M., & Friedlander, A. (2017). Algebraic procedures and creative thinking. *ZDM - Mathematics Education*, *49*(1), 53–63. https://doi.org/10.1007/s11858-016-0803-y

Takeuchi, H., & Shinno, Y. (2020). Comparing the lower secondary textbooks of Japan and England: A praxeological analysis of symmetry and transformations in geometry. *International Journal of Science and Mathematics Education*, *18*(4), 791–810. https://doi.org/10.1007/s10763-019-09982-3

Tall, D. (2004). Reflections on reserach and teavhing of equations and inequalities. *Proceedings of the 28th Conference of the International Group of Psychology of Mathematics Education*, *1*, 158–161. Bergen, Norway.

Tall, D. (2005). A theory of mathematical growth through embodiment, symbolism and proof. *International Colloquium on Mathematical Learning from Early Childhood to Adulthood*, 1–19. Nivelles, Belgium.

Tall, D. (2008a). The historical & individual developmentof mathematical thinking: Ideas that areset-before and met-before. *Plenary at HTEM conference*. Rio.

Tall, D. (2008b). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24. https://doi.org/10.1007/BF03217474

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151–169. http://dx.doi.org/ 10.1007/BF00305619

Tamba, K. P., & Saragih, M. J. (2020). Epistemological obstacles on the quadratic inequality. *Al-Jabar: Jurnal Pendidikan Matematika*, *11*(2), 317–330. https://doi.org/10.24042/ajpm.v11i2.6858

Tanisli, D., & Kose, N. Y. (2013). Preservice mathematics teachers' knowledge of students about algebraic concepts. *Australian Journal of Teacher Education*, *38*(2), 1–18. https://doi.org/10.14221/ajte.2013v38n2.1

Tanner, R. C. H. (1961). Mathematics begins with inequality. *The Mathematical Gazette*, *45*(354), 292–294. https://doi.org/10.2307/3614090

Taqiyuddin, M., Sumiaty, E., & Jupri, A. (2017). Analysis of junior high school students' attempt to solve a linear inequality problem. *AIP Conference Proceedings*, *1868*(August). https://doi.org/10.1063/1.4995160

Tavignot, P. (1995). A propos de la transposition didactique en didactique des mathématiques. *Spirale. Revue de recherches en éducation*, *15*(1), 31–60. https://doi.org/10.3406/spira.1995.1906

Tent, M. W. (2000). Teacher to Teacher: Inequalities: Part of Every Child's Life. *Mathematics Teaching in the Middle School*, *5*(5), 292–295. https://doi.org/10.5951/MTMS.5.5.0292

Trouche, L. (2016). Didactics of mathematics: Concepts, roots, interactions and dynamics from France. Dalam J. Monaghan, L. Trouche, & J. M. Borwein (Ed.), *Tools and Mathematics* (hlm. 219–256). Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-319-02396-0_10

Tsamir, P., & Almog, N. (2001). Students' strategies and difficulties: the case of algebraic inequalities. *International Journal of Mathematical Education in*

Science and Technology, *32*(4), 513–524. https://doi.org/10.1080/00207390110038277

Tsamir, Pessia, Almog, N., & Tirosh, D. (1998). Students' solutions of inequalities. *Procedings of the Conference of the International Group for the Psychology of Mathematics Education (2nd, Stelenbosch, South Africa, July 12-17, 1998.), Volume 4*, 129–136.

Tsamir, Pessia, & Bazzini, L. (2004). Consistencies and inconsistencies in students' solutions to algebraic 'single-value' inequalities. *International Journal of Mathematical Education in Science and Technology*, *35*(6), 793–812. https://doi.org/10.1080/00207390412331271357

Tsamir, Pessia, & Reshef, M. (2006). Students' preferences when solving quadratic inequalities. *Focus on Learning Problems in Mathematics*, *28*(1), 37–50.

Usiskin, Z. (1999). Conceptions of school algebra and uses of variables. Dalam B. Moses (Ed.), *In Algebraic Thinking, Grades K–12: Readings from NCTM's School-Based Journals and Other Publications*. Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Zalman. (1996). Mathematics as a language. Dalam Portia C. Elliott & Margaret J. Kenney (Ed.), *Communication in Mathematics, K–12 and Beyond Yearbook of the National Council of Teachers ofMathematics (NCTM)* (hlm. 231–243). Reston, VA: NCTM.

Vaiyavutjamai, P., & Clements, M. A. (Ken). (2006). Effects of classroom instruction on student performance on, and understanding of, linear equations and linear inequalities. *Mathematical Thinking and LearningThinking*, *8*(2), 113–147. https://doi.org/10.1207/s15327833mtl0802_2

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houng, R. T. (2003). According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks. *British Journal of Educational Studies*, *51*(4).

van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2019). *Elementary and middle school mathematics: Teaching developmentally*. USA: Pearson.

Verret, M. (1975). *Le temps des études*. Paris: Librairie Honoré Cham-pion.

Viholainen, A. (2008). Incoherence of a concept image and erroneous conclusions in the case of differentiability. *The Mathematics Enthusiast*, *5*(2), 231–248. https://doi.org/10.54870/1551-3440.1104

Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, *14*(3), 293–305. http://dx.doi.org/10.1080/0020739830140305

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. Dalam D. Tall (Ed.), *Advanced Mathematical Thinking* (Vol. 11, hlm. 65–81). Dordrecht: Springer Netherlands. https://doi.org/10.1007/0-306-47203-1_5

Warren, E. (2006). Comparative mathematical language in the elementary school: A longitudinal study. *Educational Studies in Mathematics*, *62*(2), 169–189. https://doi.org/10.1007/s10649-006-4627-5

Wijaya, A., Retnawati, H., Setyaningrum, W., Aoyama, K., & Sugiman, S. (2019). Diagnosing students' learning difficulties in the eyes of Indonesian mathematics teachers. *Journal on Mathematics Education*, *10*(3), 357–364. https://doi.org/10.22342/jme.10.3.7798.357-364

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, *89*(1), 41–65. https://doi.org/10.1007/s10649-015-9595-1

Wijayanti, D., & Winsløw, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *REDIMAT*, *6*(3), 307–330. https://doi.org/10.1783/redimat.2017.2078

Wijayanti, N., Arigiyati, T. A., Aulia, F., & Widodo, S. A. (2021). Developing of e-worksheet linear equations and inequalities based on tri-n. *Journal of Medives: Journal of Mathematics Education IKIP Veteran Semarang*, *5*(2), 245–260. https://doi.org/10.31331/medivesveteran.v5i2.1650

Willig, C. (2001). *Introducing qualitative research in psychology: Adventures in theory and method*. Buckingham, Philadelphia: Open University Press.

Winsløw, C. (2011). Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education. Dalam Marianna Bosch, J. Gascón, A. R. Olarría, M. Artaud, A. Bronner, Y. Chevallard, … M. Larguier (Ed.), *An overview of ATD* (hlm. 117–138). Barcelona: Centre de Recerca Matemàtica.

Winsløw, C., Barquero, B., de Vleeschouwer, M., & Hardy, N. (2014). An institutional approach to university mathematics education: From dual vector spaces to questioning the world. *Research in Mathematics Education*, *16*(2), 95–111. https://doi.org/10.1080/14794802.2014.918345

Yasa, N. Y. P., Astuti, D., & Sukar, S. (2022). Analyzing students' mathematical connection ability on linear inequality system in two-variable. *International Conferenceon Education*, *1*, 254–262. Yogyakarta: Faculty of Teacher Training and Education, Universitas Ahmad Dahlan.

Yenilmez, K., & Avcu, T. (2009). Primary school students' difficulties in learning absolute value. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi*, *12*, 80–88.

Yığ, K. G. (2022). Research trends in mathematics education: A quantitative content analysis of major journals 2017-2021. *Journal of Pedagogical Research*, *6*(3), 137–153. https://doi.org/10.33902/JPR.202215529

Zhou, M., & Brown, D. (2015). *Educational learning theories: 2nd edition*. Education Open Textbooks.

Zill, D. G., & Dewar, J. (2016). *Essentials of precalculus with calculus previews* (Sixth edition). Burlington, MA: Jones & Bartlett Learning.

Zulhendri, Effendi, M., & Darmansyah. (2022). Analysis of Student's Error in Solving Linear Inequality. *East Asian Journal of Multidisciplinary Research (EAJMR)*, *1*(4), 559–570.