

**REPRESENTASI TRANSISI BERPIKIR ARITMETIK KE  
BERPIKIR ALJABAR SISWA SEKOLAH MENENGAH PERTAMA**

DISERTASI

diajukan untuk memenuhi sebagian syarat memperoleh  
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PROGRAM STUDI PENDIDIKAN MATEMATIKA  
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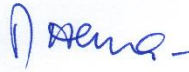
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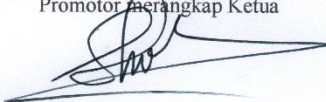
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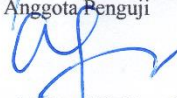
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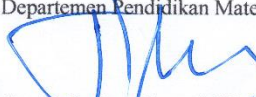


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## ABSTRAK

### **Reni Wahyuni (2023). Representasi Transisi Berpikir Aritmetik Ke Berpikir Aljabar Siswa Sekolah Menengah Pertama.**

Penelitian ini bertujuan mendeskripsikan representasi transisi berpikir aritmetik ke berpikir aljabar pada siswa kelas VII Sekolah Menengah Pertama (SMP) pada siswa kelas VII SMP yang mana siswa memiliki usia yang sama yaitu 13 tahun. Metode penelitian yang digunakan pada penelitian ini adalah pendekatan kualitatif dengan jenis penelitian fenomenologi. Tempat penelitian merupakan tiga SMP yang ada di Pekanbaru, Riau dengan total partisipan sebanyak 18 siswa. Teknik pengumpul data yang digunakan pada penelitian ini adalah tes representasi transisi berpikir aritmetik ke berpikir aljabar, wawancara siswa, dan lembar observasi berupa catatan lapangan. Temuan penelitian yang diperoleh pada penelitian ini berdasarkan pada lima karakteristik representasi transisi berpikir aritmetik ke berpikir aljabar. Secara keseluruhan siswa kelas VII SMP melakukan representasi transisi berpikir aritmetik ke berpikir aljabar berupa; (1) memahami arti simbol tanda sama dengan sebagai simbol relasional, operasional-relasional serta operasional; (2) memahami sifat komutatif dan sifat distributif masih belum cukup baik karena siswa cenderung menganggap bahwa struktur bilangan yang berkaitan dengan sifat komutatif dan sifat distributif adalah suatu hal untuk melakukan perhitungan ; (3) menggunakan operasi-operasi dasar sesuai dengan urutan operasi-operasi dasar; (4) memaknai huruf-huruf aljabar sebagai sebagai *unknown* dan *generalized numbers* sehingga ada siswa mampu untuk membentuk ekspresi aljabar dan ada yang tidak mampu membentuk ekspresi aljabar; (5) membentuk ragam representasi matematis yaitu persamaan, visual berupa gambar, tabel bilangan dan garis bilangan.

Kata kunci: representasi, transisi berpikir aritmetik ke berpikir aljabar, ide-ide aljabar

## ABSTRACT

**Reni Wahyuni (2023). The Representation of The Transition from Arithmetic to Algebraic Thinking in Junior High School Students.**

This study describes the representation of the transition from arithmetic thinking to algebraic thinking in seventh grade students of junior high school, all of whom are 13 years old. This study employed a qualitative approach with a phenomenological research design. Three junior high schools in Pekanbaru, Riau, were used as research location, with 18 students participating. This study's data collection methods included arithmetic thinking to algebraic thinking transition representation tests, student interviews, and observation sheets in the form of field notes. The research findings in this study are based on five characteristics of the representation of the transition from arithmetic thinking to algebraic thinking. Overall, seventh grade junior high school students represent the transition from arithmetic to algebraic thinking by; (1) understanding the meaning of the equal sign symbol as a relational, operational-relational, and operational symbol; (2) understanding commutative and distributive properties is still lacking because students believe that the number structure relating with commutative and distributive properties is something to be calculated with; (3) applying basic operations in accordance with the order of operations; (4) interpreting algebraic letters as unknown and generalized numbers, while some students are unable to form algebraic expressions; and (5) creating a variety of mathematical representations, such as equations, images, number tables, and number lines. Keywords: representation, the transition from arithmetic to algebraic thinking, ideas of algebra

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### DAFTAR PUSTAKA

- Amerom, B. A. Van. (2002). Reinvention of early algebra Developmental Research on The Transition from Arithmetic to Algebra. In *Thesis Utrecht University*. Utrecht University.
- Appah, M. K., Brown, I. G., & Baidoo, S. R. (2020). Algebraic Thinking among Primary Pupils: A Boost for Interest in Mathematics. *Pedagogical Research*, 5(2). <https://doi.org/10.29333/pr/7878>
- Arcavi, A., Drijvers, P., & Stacey, K. (2017). *The Learning and Teaching of Algebra Ideas, Insights, and Activities* (1st ed.). Routledge.
- As'ari, A. R., Tohir, M., Valentino, E., Imron, Z., & Taufiq, I. (2017). *Matematika SMP/MTs Kelas VII Semester 1* (A. Lukito, A. Mahmudi, Turmudi, M. N. Priatna, Y. Satria, & Widowati (eds.); Edisi Revi). Pusat Kurikulum dan Perbukuan, Balitbang, Kemendikbud.
- Bajwa, N. P., & Perry, M. (2019). Features of a pan balance that may support students' developing understanding of mathematical equivalence. *Mathematical Thinking and Learning*, 1–27. <https://doi.org/10.1080/10986065.2020.1700587>
- Bandur, A. (2019). *Penelitian Kualitatif: Studi Multi-Disiplin Keilmuan dengan Nvivo 12 Plus* (1st ed.). Mitra Wacana Media.
- Bazeley, P., & Jackson, K. (2013). *Qualitative data analysis with NVIVO* (J. Seaman (ed.); 2nd ed.). The British Library. <https://doi.org/10.1080/02607476.2013.866724>
- Bednarz, N., Kieran, C., & Lee, L. (1996). Approaches to Algebra: Perspective for Research and Teaching. In L. L. Bernarz N., Kieran C. (Ed.), *Approaches to Algebra* (Approaches, pp. 3–12). Mathematics Education Library:Springer. [https://doi.org/https://doi.org/10.1007/978-94-009-1732-3\\_1](https://doi.org/https://doi.org/10.1007/978-94-009-1732-3_1)
- Blanton, M. L., Brizuela, B. M., Gardiner, A. M., Sawrey, K., & Newman-Owens, A. (2015). A Learning Trajectory in 6-Year-Olds' Thinking About Generalizing Functional Relationships. *Journal for Research in Mathematics Education*, 46(5), 511–558. <https://doi.org/10.5951/jresematheduc.46.5.0511>
- Blanton, M. L., Brizuela, B. M., Stephens, A. C., Knuth, E. J., Isler, I., Gardiner, A. M., Stroud, R., Fonger, N. L., & Stylianou, D. (2018). Implementing a Framework for Early Algebra. In C. Kieran (Ed.), *Teaching and Learning Algebraic Thinking with 5-to-12-Year-Olds, ICME-13* (pp. 27–49). Springer International Publishing AG. [https://doi.org/https://doi.org/10.1007/978-3-319-68351-5\\_2](https://doi.org/https://doi.org/10.1007/978-3-319-68351-5_2)
- Blanton, M. L., & Kaput, J. J. (2005). Characterizing a Classroom Practice That Promotes Algebraic Reasoning. *Journal for Research in Mathematics Education*, 36(5), 412–446.
- Blanton, M. L., & Kaput, J. J. (2011). Functional Thinking as a Route Into Algebra in the Elementary Grades. In J. Cai & E. Knuth (Eds.), *Early Algebraization, Advances in Mathematics Education* (pp. 5–23). Springer-Verlag Berlin Heidelberg. <https://doi.org/10.1007/BF02655895.J.J.>

- Blanton, M. L., Stephens, A. C., Knuth, E. J., Gardiner, A. M., Isler, I., & Kim, J. (2015). The Development of Children's Algebraic Thinking: The Impact of a Comprehensive Early Algebra Intervention in Third Grade. *Journal for Research in Mathematics Education*, 46(1), 39–87.
- Booker, G., & Windsor, W. (2010). Developing Algebraic Thinking: using problem-solving to build from number and geometry in the primary school to the ideas that underpin algebra in high school and beyond. *Procedia - Social and Behavioral Sciences*, 8, 411–419. <https://doi.org/10.1016/j.sbspro.2010.12.057>
- Booth, L. R. (1988). Children's difficulties in beginning algebra. In *The ideas of algebra, K-12* (pp. 299–306). <http://elementaryalgebra.cmswiki.wikispaces.net/file/view/Childrens+Difficulties+in+Beginning+Algebra.pdf/142535729/Childrens+Difficulties+in+Beginning+Algebra.pdf%0Ahttp://elementaryalgebra.cmswiki.wikispaces.net/file/view/Childrens+Difficulties+in+Begin>
- Boulton-Lewis, G., Cooper, T. J., Atweh, B., Pillay, H., & Wills, L. (1995). Arithmetic, pre-algebra and algebra: A model of transition. *Teaching Mathematics in New Times Proceedings of the 21st Annual Conference of the Mathematics Research Group of Australasia*, 14, 114-120 ST-Arithmetic, pre-algebra and algebra:
- Bråting, K., Madej, L., & Hemmi, K. (2019). Development of algebraic thinking: opportunities offered by the Swedish curriculum and elementary mathematics textbooks. *Nordic Studies in Mathematics Education*, 24(1), 27–49.
- Britt, M. S., & Irwin, K. C. (2011). Algebraic Thinking with and without Algebraic Representation: A Pathway for Learning. In J. Cai & E. Knuth (Eds.), *Early Algebraization, Advances in Mathematics Education* (pp. 137–159). Springer-Verlag Berlin Heidelberg. <https://doi.org/10.1007/978-3-642-17735-4>
- Byrd, C. E., McNeil, N. M., Chesney, D. L., & Matthews, P. G. (2015). A specific misconception of the equal sign acts as a barrier to children's learning of early algebra. *Learning and Individual Differences*, 38, 61–67. <https://doi.org/10.1016/j.lindif.2015.01.001>
- Cai, J., & Knuth, E. J. (2011). Early Algebraization: A Global Dialogue from Multiple Perspectives. In G. Kaiser & B. Sriraman (Eds.), *Advances in Mathematics Education*. Springer. <https://doi.org/10.1007/978-3642-17735-4>
- Cai, J., & Moyer, J. C. (2006). Developing Algebraic Thinking in Earlier Grades: Some Insights from International Comparative Studies. *The Nature of Algebra and Algebraic Thinking: Content Perspective*.
- Cai, J., Ng, S. F., & Moyer, J. C. (2011). Developing Students' Algebraic Thinking in Earlier Grades: Lessons from China and Singapore. In *Early Algebraization: A Global Dialogue from Multiple Perspectives*. Springer.
- Carraher, D., & Schliemann, A. (2016). Powerful Ideas in Elementary School Mathematics. In L. D. English & D. Kirshner (Eds.), *Handbook of International Research in Mathematics Education* (pp. 191–218). Routledge.

- Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Darrell, E. (2006). Arithmetic and Algebra in Early Mathematics Education. *Journal for Research in Mathematics Education*, 37(2), 87–115. <https://doi.org/10.2307/30034843>
- Carraher, D. W., & Schliemann, D. (2018). Cultivating Early Algebraic Thinking. In C. Kieran (Ed.), *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds, ICME-13 Monographs* (pp. 107–138). Springer International Publishing AG. [https://doi.org/https://doi.org/10.1007/978-3-319-68351-5\\_5](https://doi.org/https://doi.org/10.1007/978-3-319-68351-5_5)
- Chaurasia, P. (2016). Algebraic Reasoning at Elementary Level : Filling the Gaps Between Arithmetic and Algebra. *International Journal of Scientific Research*, 5(10), 240–244.
- Chimoni, M., Pitta-Pantazi, D., & Christou, C. (2018). Examining early algebraic thinking: insights from empirical data. *Educational Studies in Mathematics*, 98(1), 57–76. <https://doi.org/10.1007/s10649-018-9803-x>
- Chow, T.-C. F. (2011). Students' Difficulties, Conceptions, and Attitudes towards Learning Algebra : An Intervention Study to Improve Teaching and Learning [Curtin University]. In A *Dissertation*. <http://hdl.handle.net/20.500.11937/1385>
- Christou, K. P., & Vosniadou, S. (2012). What Kinds of Numbers Do Students Assign to Literal Symbols? Aspects of the Transition from Arithmetic to Algebra. *Mathematical Thinking and Learning*, 14, 1–27. <https://doi.org/10.1080/10986065.2012.625074>
- Creswell, J. W. (2012). *Educational Research Planning, Conducting and Evaluating Quantitative and Qualitative Research* (P. A. Smith (ed.); 4th ed.). Pearson Education, Inc.
- Creswell, J. W. (2014). *Research Design Pendekatan Metode Kualitatif, Kuantitatif, dan Campuran* (A. Fawaid & R. K. Pancasari (trans.); Edisi Keem). Pustaka Pelajar.
- Dahlan, J. A. (2011). *Materi Pokok Analisis Kurikulum Matematika* (1st ed.). Universitas Terbuka.
- Dahlan, J. A., & Juandi, D. (2011). Analisis Representasi Matematik Siswa Sekolah Dasar Dalam Penyelesaian Masalah Matematika Kontekstual. *Jurnal Pengajaran Matematika Dan Ilmu Pengetahuan Alam*, 16(1), 128. <https://doi.org/10.18269/jpmipa.v16i1.273>
- Darhim. (n.d.). *Permainan matematika sebagai latihan untuk menumbuhkan minat terhadap matematika*.
- Dekker, T., & Dolk, M. (2011). From Arithmetic To Algebra. In P. Drijvers (Ed.), *Secondary Algebra Education* (Issue 1927, pp. 69–87). Sense Publishers. <https://doi.org/10.4324/9781315097435-4>
- Drijvers, P. (2003). Learning Algebra in A Computer Algebra Environment: Design Research on The Understanding of The Concept of Parameter. In *Dissertation*. Utrecht University.
- Driscoll, M. (1999). Developing Algebraic Habits of Mind: A Framework for Classroom Questions Aimed at Understanding Student Thinking. In *Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10*. (pp. 1–

- 19). Heinemann.  
<http://www.eric.ed.gov/ERICWebPortal/recordDetail?accno=ED465556>
- Ely, R., & Adams, A. E. (2012). Unknown, placeholder, or variable: What is x? *Mathematics Education Research Journal*, 24(1), 19–38. <https://doi.org/10.1007/s13394-011-0029-9>
- Ertekin, E. (2017). Predicting Eight Grade Students' Equation Solving Performances via Concepts of Variable and Equality. *Journal of Education and Practice*, 8(21), 74–80.
- Fong, N. S. (2004). Developing Algebraic Thinking in Early Grades : Case Study of the Singapore Primary Mathematics Curriculum. *The Mathematics Educator*, 8(1), 39–59.
- Fulop, Z. (2015). Transition from arithmetic to algebra in primary school education. *Teaching Mathematics and Computer Science*, 2, 225–248. <https://doi.org/10.5485/TMCS.2015.0398>
- Goldin, G. A. (2020). Mathematical Representations. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 566–572). Springer International Publishing. [https://doi.org/10.1007/978-3-030-15789-0\\_103](https://doi.org/10.1007/978-3-030-15789-0_103)
- Goldin, G. A., & Kaput, J. J. (1996). A Joint Perspective on the Idea of Representation in Learning and Doing Mathematics. *Theories of Mathematical Learning*, September, 397–430. <https://doi.org/10.4324/9780203053126-30>
- González, M. M., Ambrose, R., & Martínez, E. C. (2004). In the transition from arithmetic to algebra: Misconceptions of the equal sign. *28th International Group for the Psychology of Mathematics Education, July 14-18, 2004, 2003*, 1–8.
- Hancock, B., Ockleford, E., & Windridge, K. (2009). An introduction to qualitative research. In *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. Second Ed.* (Vol. 4th). The NIHR RDS for the East Midlands. <https://doi.org/10.1109/TVCG.2007.70541>
- Hatch, J. A. (2002). *Doing Qualitative Research in Education Settings*. State University of New York Press.
- Headlam, C., & Graham, T. (2009). Some initial findings from a study of children's understanding of the Order of Operations. *Proceedings of the British Society for Research into Learning Mathematics*, 29(3), 37–42.
- Hendriana, H., Rohaeti, E. E., & Sumarmo, U. (2017). *Hard Skills dan Soft Skills* (N. F. Atif (ed.)). PT Refika Aditama.
- Herawati, E., & Kadarisma, G. (2021). Analisis Kesulitan Siswa SMP Kelas VII dalam Menyelesaikan Soal Operasi Aljabar. *Jurnal Pembelajaran Matematika Inovatif*, 4(2), 355–364. <https://doi.org/10.22460/jpmi.v4i2.355-364>
- Herman, T. (2007). Pembelajaran Matematika Berbasis Masalah Untuk Meningkatkan Kemampuan Berpikir Tingkat Tinggi Siswa SLTP. *Educationist*, 1(1), 1–20.
- Herscovics, N., & Linchevski, L. (1994). A Cognitive Gap Between Arithmetic And Algebra. In *Educational Studies in Mathematics* (Vol. 27, pp. 59–78). Kluwer Academic Publishers.

- Herutomo, R. (2017). Miskonsepsi Aljabar : Konteks Pembelajaran Matematika pada Siswa Kelas VIII SMP. *Journal Of Basication: Jurnal Pendidikan Dasar*, 1(November), 1–8.
- Hidayanto, E. (2013). Proses Berpikir Aritmetika dan Berpikir Aljabar Siswa dalam Menyelesaikan Soal Cerita. *Prosiding Seminar Nasional Aljabar Dan Pembelajarana*, 20 April 2013, 173–177.
- Hidayanto, E., Purwanto, Subanji, & Rahardjo, S. (2014). *Transisi dari berpikir aritmetis ke berpikir aljabaris*. July. <https://www.researchgate.net/publication/275031674>
- Hitt, F., Cortés, C., & Saboya, M. (2017). Integrating Arithmetic and Algebra in a Collaborative Learning and Computational Environment Using ACODESA. In G. Aldon, F. Hitt, L. Bazzini, & U. Gellert (Eds.), *Mathematics and Technology, Advances in Mathematics Education* (pp. 285–311). Springer International Publishing. [https://doi.org/10.1007/978-3-319-51380-5\\_13](https://doi.org/10.1007/978-3-319-51380-5_13)
- Hunter, J. (2010). Developing early algebraic reasoning through exploration of the commutative principle. In M. Joubert & P. Andrews (Eds.), *Proceedings of the British Congress for Mathematics Education* (Issue April, pp. 105–112).
- Jupri, A., & Drijvers, P. (2016). Student Difficulties in Mathematizing Word Problems in Algebra. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(9), 2481–2502. <https://doi.org/10.12973/eurasia.2016.1299a>
- Jupri, A., Drijvers, P., & Heuvel-Panhuizen, M. van den. (2014). Difficulties in Initial Algebra Learning in Indonesia. *Mathematics Education Research Journal*, 26(4), 683–710. <https://doi.org/10.1007/s13394-013-0097-0>
- Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). The impact of a technology-rich intervention on grade 7 students' skills in initial algebra. *Innovation and Technology for Mathematics Education: Proceedings of the 19th Asian Technology Conference in Mathematics*, 152–161.
- Kaput, J. J. (2008). What Is Algebra? What is Algebraic Reasoning? In J. J. Kaput, D. W. Carragher, & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 8–17). Taylor&Francis Group, LLC.
- Kartini. (2009). Peranan Representasi dalam Pembelajaran Matematika. *Seminar Nasional Matematika Dan Pendidikan Matematika UNY*, 361–372.
- Khalid, M., Yakop, F., & Ibrahim, H. (2020). Year 7 students' interpretation of letters and symbols in solving routine algebraic problems. *Qualitative Report*, 25(11), 4167–4181. <https://doi.org/10.46743/2160-3715/2020.4062>
- Kieran, C. (2004). Algebraic Thinking in The Early Grades : What is it? *The Mathematics Educator*, 8(1), 139–151. <https://www.researchgate.net/publication/228526202%0AAlgebraic>
- Kieran, C. (2018). *Teaching and Learning Algebraic Thinking with 5-to 12-Year-Olds The Global Evolution of an Emerging Field of Research and Practice* (G. Kaiser (ed.)). Springer International Publishing AG. <http://www.springer.com/series/15585>
- Kieran, C., & Chalouh, L. (1993). Prealgebra : The Transition from Arithmetic to

- Algebra. In D. T. Owens (Ed.), *Research ideas for the classroom: Middle grades mathematics* (pp. 179–198). Macmillan Publishing Company.
- Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). *Early Algebra Research into its Nature, its Learning, its Teaching* (G. Kaiser (ed.)).
- Kilhamn, C. (2014). When Does a Variable Vary? Identifying Mathematical Content Knowledge for Teaching Variables. *Nordic Studies in Mathematics Education*, 19(3–4), 83–100.
- Kilhamn, C., Røj-Lindberg, A.-S., & Björkqvist, O. (2019). School Algebra. In C. Kilhamn & R. Saljo (Eds.), *Encountering Algebra* (pp. 1–12). Springer International Publishing Switzerland. <https://doi.org/10.1007/978-3-030-17577-1>
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics* (J. Kilpatrick, J. Swafford, & B. Findell (eds.); Mathematic). National Academy Press. <https://doi.org/10.17226/9822>
- Kızıltoprak, A., & Köse, N. Y. (2017). Relational thinking: The bridge between arithmetic and algebra. *International Electronic Journal of Elementary Education*, 10(1), 131–145. <https://doi.org/10.26822/iejee.2017131893>
- Knuth, E. J., Alibali, M. W., Hattikudur, S., Mcneil, N. M., & Stephens, A. C. (2008). The Importance of Equal Sign Understanding in The Middle Grades. *Mathematics Teaching in the Middle School*, 13(9), 514–519.
- Knuth, E. J., Stephens, A. C., Blanton, M. L., & Fardiner, A. (2016). Build an early foundation for algebra success. *Kappan*, 97(6), 65–68.
- Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does Understanding the Equal Sign Matter? Evidence from Solving Equations. *Journal for Research in Mathematics Education*, 37(4), 297–312.
- Kriegler, S. (2007). Just What is Algebraic Thinking? In *Algebraic Concepts in the Middle School*.
- Kusumayanti, R. (2021). Peran PPPPTK Matematika Dalam Meningkatkan Capaian PISA - Artikel. In *PPPPTK Matematika*.
- Lee, J., & Pang, J. S. (2020). Students' opposing conceptions of equations with two equal signs. *Mathematical Thinking and Learning*, 00(00), 1–16. <https://doi.org/10.1080/10986065.2020.1777364>
- Linchevski, L., & Livneh, D. (1999). Structure Sense: The Relationship between Algebraic and Numerical Contexts. *Educational Studies in Mathematics*, 40(2), 173–196. <https://doi.org/10.1023/A:1003606308064>
- Lincoln, Y., & Guba, E. (1985). Establishing Trustworthiness. In *Naturalistic Inquiry* (pp. 289–327). SAGE. <https://doi.org/10.4337/9781800371323.00016>
- Macgregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11–15. *Educational Studies in Mathematics*, 33, 1–19.
- Maffia, A., & Mariotti, M. A. (2017). Seeking symmetry in distributive property : Children developing structure sense in arithmetic. *CERME 10*, 01873466.
- Mahmudi, A. (2016). Memberdayakan Pembelajaran Matematika Untuk Mengembangkan Kompetensi Masa Depan. *Seminar Nasional Matematika Dan Pendidikan Matematika UNY*, 1–6. <http://seminar.uny.ac.id/semnasmatematika/sites/seminar.uny.ac.id.semnas>

matematika/files/U-1.pdf

- Malisani, E., & Spagnolo, F. (2009). From Arithmetical Thought to Algebraic Thought : The Role of The“Variable”. *Educational Studies in Mathematics*, 71, 19–41. <https://doi.org/10.1007/s10649-008-9157-x>
- Mashazi, S. (2013). Learners’ Explanations of The Errors They Make in Introductory Algebra. *Pythagoras*, 9, 14–33.
- Matthews, P. G., & Fuchs, L. S. (2018). Keys to the Gate? Equal Sign Knowledge at Second Grade Predicts Fourth-Grade Algebra Competence. *Child Development*, 91(1), 1–15. <https://doi.org/10.1111/cdev.13144>
- Matthews, P. G., Rittle-Johnson, B., McEldoon, K. L., & Taylor, R. (2012). Measure for Measure: What Combining Diverse Measures Reveals About Children’s Understanding of the Equal Sign as an Indicator of Mathematical Equality. *Journal for Research in Mathematics Education*, 43(3), 316–350. <https://doi.org/10.5951/jresmetheduc.43.3.0316>
- McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. E. (2006). Middle-school Students’ Understanding of the Equal sign: The Books They Read Can’t Help. *Cognition and Instruction*, 24(3), 367–385. [https://doi.org/10.1207/s1532690xci2403\\_3](https://doi.org/10.1207/s1532690xci2403_3)
- Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative Research: A Guide to Design and Implementation* (Fourth). Jossey-Bass.
- Miles, M. B., Huberman, A. M., & Saldana, H. (2014). *Qualitative Data Analysis: A Methods Sourcebook* (3rd ed.). Arizona State University. <https://www.ptonline.com/articles/how-to-get-better-mfi-results>
- Molina, M., Ambrose, R., & Rio, A. (2018). First Encounter with Variables by First and Third Grade Spanish Students. *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds: The Global Evolution of an Emerging Field of Research and Practice, 2017*, 261–280.
- NCTM. (2000). *Principles and Standards for School Mathematics*. The National Council of Teachers Mathematics.
- Nursupriana, I., & Nisa, N. H. (2013). Pengaruh Pemahaman Konsep Aritmatika Terhadap Kemampuan Berpikir Aljabar Siswa (Studi Kasus pada Siswa Kelas VII SMP Negeri 1 Ketanggungan Kabupaten Brebes ). *Eduma : Mathematics Education Learning and Teaching*, 2(2). <https://doi.org/10.24235/eduma.v2i2.39>
- Olteanu, L. (2017). Distributive law as object of learning through direct and inverse tasks. *International Journal for Lesson and Learning Studies*, 6(1), 56–65. <https://doi.org/10.1108/IJLLS-05-2016-0014>
- Pittalis, M., Pitta-pantazi, D., & Christou, C. (2018). A Longitudinal Study Revisiting the Notion of Early Number Sense : Algebraic Arithmetic AS a Catalyst for Number Sense Development A Longitudinal Study Revisiting the Notion of Early Number Sense : Algebraic Arithmetic AS a Catalyst for Number Sense Devel. *Mathematical Thinking and Learning*, 20(3), 222–247. <https://doi.org/10.1080/10986065.2018.1474533>
- Powell, S. R., Kearns, D. M., & Driver, M. K. (2016). Exploring the connection between arithmetic and prealgebraic reasoning at first and second grade.

- Journal of Educational Psychology*, 108(7), 943–959.  
<https://doi.org/10.1037/edu0000112>
- Pratiwi, I. (2019). Efek Program Pisa Terhadap Kurikulum Di Indonesia. *Jurnal Pendidikan Dan Kebudayaan*, 4(1), 51–71.  
<https://doi.org/10.24832/jpnk.v4i1.1157>
- Radford, L. (2006). Algebraic Thinking and The Generalization of Patterns: A Semiotic Perspective. In S. Alatorre, J. L. Cortina, M. Sáiz, & A. Méndez (Eds.), *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. (Vol. 1, pp. 2–21). Universidad Pedagógica Nacional.
- Radford, L. (2018). The Emergence of Symbolic Algebraic Thinking in Primary in Primary School. In C. Kieran (Ed.), *Teaching and Learning Algebraic Thinking with 5-to-12-Year-Olds: The Global Evolution of an Emerging Field of Research and Practice* (Issue January, pp. 3–25). Springer.  
<https://doi.org/10.1007/978-3-319-68351-5>
- Rahmawati, D., Purwanto, Subanji, Hidayanto, E., & Anwar, R. B. (2017). Process of Mathematical Representation Translation from Verbal into Graphic. *International Electronic Journal of Mathematics Education*, 12(3), 367–381. <https://doi.org/10.29333/iejme/618>
- Rézio, A. S. R. (2014). Algebraic Thinking: Conceptions of Elementary School Teachers. *Revista Ensayos Pedagógicos*, 9(1), 121–135.  
<https://doi.org/10.15359/rep.9-1.6>
- Rhine, S., Harrington, R., & Starr, C. (2019). *How Students Think When Doing Algebra*. Information Age Publishing.
- Riinawati. (2021). *Pengantar Evaluasi Pendidikan*. Thema Publishing.
- Rosyida, A. N. (2019). *Aritmatika* (Y. Winarti (ed.)). ALPRIN.
- Roy, S., & Roth, D. (2015). Solving general arithmetic word problems. *Conference Proceedings - EMNLP 2015: Conference on Empirical Methods in Natural Language Processing*, 1743–1752.  
<https://doi.org/10.18653/v1/d15-1202>
- Rystedt, E. (2015). *Encountering algebraic letters, expressions and equations: A study of small group discussions in a Grade 6 classroom* [The University of Gothenburg]. <http://hdl.handle.net/2077/40140>
- Sabirin, M. (2014). Representasi Dalam Pembelajaran Matematika. *JPM IAIN Antasari*, 1(2), 33–44.
- Sadovsky, P., & Sessa, C. (2005). The didactic interaction with the procedures of peers in the transition from arithmetic to algebra: A milieu for the emergence of new questions. *Educational Studies in Mathematics*, 59(1–3), 85–112. <https://doi.org/10.1007/s10649-005-5886-2>
- Samsuddin, A. F., & Retnawati, H. (2018). Mathematical representation: the roles, challenges and implication on instruction. *Journal of Physics: Conference Series*, 1097(1), 012152. <https://doi.org/10.1088/1742-6596/1097/1/012152>
- Santarone, D., Abney, A. R., & Samples, B. (2020). Heading toward Equality: Preservice Teachers' Interventions to Change Students' Conceptions of the Equal Sign. *Investigations in Mathematics Learning*, 12(3), 208–225.  
<https://doi.org/10.1080/19477503.2020.1781348>



- Sari, A. F., & Ernawati, A. (2019). Arithmetic thinking : Reflect on the order of operation. *Proceedings of The Second AHmad Dahlan International Conference on Mathematics and Mathematics Education*, 2(1), 13–18.
- Sari, H. M., & Afriansyah, E. A. (2020). Analisis Miskonsepsi Siswa SMP pada Materi Operasi Hitung Bentuk Aljabar. *Mosharafa: Jurnal Pendidikan Matematika*, 9(3), 439–450. <https://doi.org/10.31980/mosharafa.v9i3.511>
- Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls of reification - The case of algebra. *Educational Studies in Mathematics*, 26(2–3), 191–228. <https://doi.org/10.1007/BF01273663>
- Slavit, D. (1999). The Role of Operation Sense in Transition from Arithmetic to Algebraic Thought. *Educational Studies in Mathematics*, 37, 251–274. <https://doi.org/10.1023/A>
- Smith, J. P., & Thompson, P. W. (2007). Quantitative Reasoning and the Development of Algebraic Reasoning. In J. J. Kaput, D. W. Carragher, & M. L. Blanton (Eds.), *Algebra In The Early Grades* (pp. 95–132). Erlbaum. <https://doi.org/10.4324/9781315097435-5>
- Solso, R. (2008). *Psikologi Kognitif* (8th ed.). Erlangga.
- Stacey, K., & Chick, H. (2004). Solving the Problem with Algebra. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The Future of the Teaching and Learning of Algebra The 12th ICMI Study* (pp. 1–20). Kluwer Academic Publishers.
- Stacey, K., & Macgregor, M. (2001). Curriculum Reform and Approaches to Algebra. In R. Sutherland, T. Rojano, A. Bell, & R. Lins (Eds.), *Perspectives on School Algebra* (Vol. 22, pp. 141–153). Springer. [https://doi.org/10.1007/0-306-47223-6\\_8](https://doi.org/10.1007/0-306-47223-6_8)
- Steinweg, A. S. (2017). Making implicit potential explicit – key ideas of algebraic thinking in primary school. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education-CERME 10*. Institute of Education, Dublin City University. [https://keynote.conference-services.net/resources/444/5118/pdf/CERME10\\_0052.pdf](https://keynote.conference-services.net/resources/444/5118/pdf/CERME10_0052.pdf)
- Stephens, A. C., Fonger, N., Strachota, S., Isler, I., Blanton, M. L., Knuth, E. J., Gardiner, A. M., Stephens, A. C., Fonger, N., Strachota, S., & Isler, I. (2017). A Learning Progression for Elementary Students ' Functional Thinking A Learning Progression for Elementary Students ' Functional. *Mathematical Thinking and Learning*, 19(3), 143–166. <https://doi.org/10.1080/10986065.2017.1328636>
- Stephens, A. C., Knuth, E. J., Blanton, M. L., Isler, I., Gardiner, A. M., & Marum, T. (2013). Equation Structure and the Meaning of the Equal Sign: The impact of Task Selection in Eliciting Elementary Students' Understandings. *The Journal of Mathematical Behavior*, 32, 173–182. <https://doi.org/http://dx.doi.org/10.1016/j.jmathb.2013.02.001>
- Sugiman, Sumadyono, & Marfuah. (2016). *Karakteristik Siswa SMP dan Bilangan* (N. Priatna, Y. Anggraena, & S. Wardhani (eds.)). Direktorat Jendral Guru dan Tenaga Kependidikan Kementerian Pendidikan dan Kebudayaan.
- Sugiyono. (2014). *Memahami Penelitian Kualitatif*. CV Alfabeta.

- Suherman, E., Turmudi, Suryadi, D., Herman, T., Suhendra, Prabawanto, S., Nurjanah, & Rohayati, A. (2003). *Strategi Pembelajaran Matematika Kontemporer* (Revisi). Jurusan Pendidikan Matematika FPMIPA UPI.
- Sujanto, A. (2009). *Psikologi Umum*. Bumi Aksara.
- Sumaryanta, Priatna, N., & Sugiman. (2019). Pemetaan Hasil Ujian Nasional Matematika. *Indonesian Digital Journal of Mathematics and Education*, 6(1), 543–557. <http://p4tkmatematika.kemdikbud.go.id/journals/index.php/idealmathedu/>
- Suryadi, D. (2010). Penelitian Pembelajaran Matematika Untuk Pembentukan Karakter Bangsa. *Seminar Nasional Matematika Dan Pendidikan Matematika Yogyakarta, 1*(November), 1–14.
- Suryadi, D. (2011). Pendidikan Matematika. -, 1–38.
- Tabak, S. (2019). 6th, 7th and 8th Grade Students' Misconceptions about the Order of Operations. *International Journal of Educational Methodology*, 5(3), 363–373. <https://doi.org/10.12973/ijem.5.3.363>
- Tall, D. (1992). The Transition from Arithmetic to Algebra : Number Patterns , or Proceptual Programming ? In *From Numeracy to Algebra*.
- Tent, M. W. (2006). Understanding the Properties of Arithmetic: A Prerequisite for Success in Algebra. *Mathematics Teaching in the Middle School*, 12(1), 22–25. <https://doi.org/http://www.jstor.org/stable/41164018>
- Usiskin, Z. (1999). Conceptions of school algebra and uses of variables. In B. Moses (Ed.), *The Ideas of Algebra, K–12. 1988 Yearbook of the National Council of Teachers of Mathematics* (A.F Coxfor, Issue 5, pp. 7–13). National Council of Teachers of Mathematics.
- Vermeulen, C., & Meyer, B. (2017). The Equal Sign : Teachers ' Knowledge and Students ' Misconceptions. *African Journal of Research in Mathematics, Science, and Technology Education*, 1–12. <https://doi.org/10.1080/18117295.2017.1321343>
- Villegas, J. L., Castro, E., & Gutiérrez, J. (2009). Representations in problem solving: A case study with optimization problems. *Electronic Journal of Research in Educational Psychology*, 7(17), 279–308.
- Wahyuni, R., & Herman, T. (2019). Is Algebraic Thinking Suitable for Indonesia Elementary School Curriculum? *Advance in Social Science, Education and Humanities Research, 3rd Asian Education Symposium, 253*(AES 2018), 77–80. <https://doi.org/10.2991/aes-18.2019.18>
- Wahyuni, R., Prabawanto, S., & Herman, T. (2020). Students' difficulties in solving algebra task in middle school. *Journal of Physics: Conference Series*, 1521(3). <https://doi.org/10.1088/1742-6596/1521/3/032071>
- Walizer, M. H., & Paul L. Wienir. (1990). *Research Methods and Analysis: Searching for Relationships* (A. S. Sadiman & S. Hutagaol (trans.); Jilid 1). Erla.
- Wang, X. (2015). The Literature Review of Algebra Learning: Focusing on the Contributions to Students' Difficulties. *Creative Education*, 6(February), 144–153. <https://doi.org/10.4236/ce.2015.62013>
- Wardhani, S. (2016). *Kurikulum matematika 2 dan aljabar 2* (A. Dhoruri (ed.)). Direktorat Jendral Guru dan Tenaga Kependidikan Kementerian Pendidikan

dan Kebudayaan.

- Warren, E. (2003). The Role of Arithmetic Structure in the Transition from Arithmetic to Algebra. *Mathematics Education Research Journal*, 15(2), 122–137.
- Warren, E., Trigueros, M., & Ursini, S. (2016). Research on the learning and teaching of Algebra. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues* (pp. 73–108). Sense Publishers. [https://doi.org/10.1007/978-94-6300-561-6\\_3](https://doi.org/10.1007/978-94-6300-561-6_3)
- Wiryanto. (2014). Representasi Siswa Sekolah Dasar dalam Pemahaman Konsep Pecahan. *Jurnal Pendidikan Teknik Elektro*, 3(3), 593–603.
- Witzel, B. S. (2016). Students With Math Difficulties and The Arithmetic to Algebra Gap. In B. S. Witzel (Ed.), *Bridging the Gap Between Arithmetic & Algebra*. Sheridan Books, Inc.