PROSES BERPIKIR MATEMATIS DAN BERPIKIR KRITIS SISWA DALAM MENYELESAIKAN MASALAH MATEMATIS NON RUTIN BERDASARKAN KERANGKA TEORI SITUASI DIDAKTIS

DISERTASI

Diajukan Sebagai Sebagian Syarat untuk Memperoleh Gelar
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ABSTRAK

Berpikir matematis dan menggunakan untuk menyelesaikan masalah adalah tujuan penting dari belajar matematika di sekolah. Oleh karena itu, penelitian yang dilakukan bertujuan untuk memaknai dan mendeskripsikan proses berpikir matematis dan berpikir kritis siswa dalam menyelesaikan masalah matematis non rutin dalam kerangka teori situasi didaktis. Metode penelitian yang digunakan adalah penelitian kualitatif dengan pendekatan fenomenologi *hermeneutik*. Tahapan analisis dilakukan dengan tiga tahap, yaitu tahap prospektif, metapedadidaktik, dan retrospektif. Partisipan yang terlibat dalam penelitian sebanyak 46 siswa dari salah satu MTs Negeri di Banda Aceh. Pada tahap analisis retrospektif diperoleh temuan tiga kategori siswa dalam menyelesaikan masalah matematis non rutin yaitu: pertama, kategori menyelesaikan masalah dengan *critical refelction*, kedua menyelesaikan masalah dengan cara *explicit reflection*, dan ketiga kategori siswa yang tidak dapat menyelesaikan masalah matematis. Kedua, situasi didaktis siswa *critical reflection* dapat memahami kedua masalah pada situasi aksi, merencanakan dan melaksanakan penyelesaian dalam situasi formulasi dan memeriksa penyelesaian pada situasi validasi; siswa *explicit reflection* memahami bilangan rasional pada situasi validasi dan masalah pola bilangan pada situasi aksi, merencanakan penyelesaian pada situasi formulasi, melaksanakan penyelesaian dan memeriksa hasil penyelesaian masalah pada situasi validasi. Sedangkan siswa yang tidak dapat menyelesaikan masalah hanya dapat memahami masalah bilangan rasional pada situasi validasi dan masalah pola bilangan pada situasi aksi. Ketiga, proses berpikir matematis: Siswa critical reflection adalah spesialisasi dengan contoh, generalisasi dalam bentuk matematika, melakukan dugaan bentuk Bahasa dan matematika dan meyakinkan dalam bentuk aljabar. Siswa *explicit refelction*, spesialisasi dengan contoh, generalisasi dalam bentuk bahasa, melakukan dugaan bentuk bahasa atau matematika dan meyakinkan dalam bentuk aritmetik. Sedangkan siswa tidak dapat menyelesaikan masalah hanya dapat melakukan specialisasi dengan memberi contoh. Ketiga, WoT siswa *critical refelction* menentukan apa yang diketahui, ditanyakan, dan diperlukan; menentukan nilai khusus atau contoh; menentukan pola; dan menggunakan masalah yang mirip. siswa *explicit refletion* menentukan apa yang diketahui, ditanyakan, dan diperlukan; menentukan nilai khusus atau contoh; menentukan pola. Sedangkan siswa tidak menyelesaikan masalah hanya menggunakan pendekatan menentukan nilai khusu atau contoh. Keempat, proses berpikir siswa *critical* dan *explicit refletion* dalam menyelesaikan masalah semua indikator Ennis dengan strategi mengklarifikasi dan menganalisis kata dan frasa; mengidentifikasi fakta yang relevan; memeriksa alasan; menyempurnakan generalisasi, mengidentifikasi alternatif penyelesaian, mengklarifikasi kata/konsep yang digunakan; dan memeriksa hasil pekerjaan. Sedangkan proses berpikir kritis yang tidak dapat menyelesaikan masalah hanya dapat mengidentifikasi fokus pertanyaan dengan strategi mengklarifikasi dan menganalisis kata dan frasa dari masalah. Kelima, *learning obstacle* yang dialami siswa yaitu *ontogenic obstacle, didactical obstacle,* dan *epistemonologis obstacle*.

Kata Kunci: Berpikir Matematis, Berpikir Kritis, Situasi Didaktis, Way of Thinking, Learning Obstacle

THE PROCESS OF STUDENTS' MATHEMATICAL THINKING AND CRITICAL THINKING IN SOLVING NON-ROUTINE MATHEMATICS PROBLEMS BASED ON A DIDACTICAL SITUATION THEORETICAL FRAMEWORK

ABSTRACT

Thinking mathematically and using it to solve problems is an essential goal of learning mathematics at school. Therefore, the research aims to interpret and describe students' mathematical and critical thinking processes in solving non-routine mathematical problems within the framework of didactic situation theory. The research method used is qualitative research with a hermeneutic phenomenological approach. The stages of analysis were carried out in three stages, namely the prospective, metapedidactical, and retrospective stages. The participants involved in the study were 46 students from one of the State MTs in Banda Aceh. In the retrospective analysis stage, it was found that three categories of students were able to solve non-routine mathematical problems, namely: first, the category of solving problems by critical reflection, the second, solving problems by means of explicit reflection, and the third category of students who could not solve mathematical problems. Second, the didactic situation of critical reflection students can understand both problems in action situations, plan and carry out solutions in formulation situations and check solutions in validation situations; Explicit reflection students understand rational numbers in validation situations and problem number patterns in action situations, plan solutions in formulation situations, carry out solutions and examine the results of solving problems in validation situations. Whereas students who cannot solve problems can only understand rational number problems in validation situations and number pattern problems in action situations. Third, the process of thinking mathematically: Critical reflection students are specializing with examples, generalizing in mathematical form, making predictions in language and mathematical forms and convincing in algebraic forms. Students are explicit reflections, specializing with examples, generalizing in linguistic forms, making predictions in linguistic or mathematical forms and convincing in arithmetic forms. While students cannot solve problems, they can only specialize by giving examples. Third, critical reflection students' WoT determines what is known, asked, and needed; specify a special value or instance; define patterns; and using similar problem. explicit reflection students determine what is known, asked, and needed; specify a special value or instance; determine pattern. While students do not solve problems only use the approach of determining special values or examples. Fourth, students' critical thinking processes and explicit reflection in solving the problems of all Ennis indicators with strategies of clarifying and analyzing words and phrases; identify relevant facts; check reasons; improve generalizations, identify alternative solutions, clarify words/concepts used; and check the results of the work. Meanwhile, critical thinking processes that cannot solve problems can only identify the focus of the question with a strategy of clarifying and analyzing the words and phrases of the problem. Fifth, the learning obstacles experienced by students are ontogenics obstacles, didactical obstacles, and epistemological obstacles.

Keywords: Mathematical Thinking, Critical Thinking, Didactic Situations, Way of Thinking, Learning Obstacles

DAFTAR ISI

REFERENSI

Abdurrahman, M. S., Abdullah, A. H., Shamsuddin, N. S., Rahman, S. N. S. A., Ashari, Z. M., Jumaat, N. F., Samah, N. A. S., & Ali, D. F. (2020). Pupil's Behaviour Pattern and Non-Routine Mathematical Problem-Solving Strategy Based on Multiple Intelligences. *Journal of Advanced Research in Dynamical and Control Systems*, *12*(3), 466–485. https://doi.org/10.5373/JARDCS/V12I3/20201214

Adorno, T. W. (1998). *Critical Models: Interventions and Catchwords* (Vol. 1). Columbia University Press.

Aiyub, A., Suryadi, D., Fatimah, S., & Kusnandi, K. (2021). Investigation of Watson-Glaser Critical Thinking Skills of Junior High School Students in Solving Mathematical Problems. *Journal of Physics: Conference Series*, *1806*(1), 012090. https://doi.org/10.1088/1742-6596/1806/1/012090

Aiyub, A., Suryadi, D., Fatimah, S., & Kusnandi, K. (2022). Investigation of the Critical Thinking Process in Solving Non-Routine Mathematical Problems. *European Online Journal of Natural and Social Sciences*, *11*(4), 1212–1233.

Alkan Ve, & Bukova Güzel. (2005). Development of Mathematical Thinking in the Student Teachers. *Gazi Education Faculty Journal*, *25*(3), 221–236.

Andrade, R. R., & Pasia, A. E. (2020). Mathematical creativity of pre-service teachers in solving non-routine problems in State University in Laguna. *Universal Journal of Educational Research*, *8*(10), 4555–4567. https://doi.org/10.13189/ujer.2020.081024

Arslan, S., & Yildiz, C. (2010). Reflections from the Experiences of 11th Graders

during the Stages of Mathematical Thinking. *Education and Science*, *35*(156), 17–31.

As'ari, A. R. (2014). Ideas for Developing Critical Thinking at Primary School Level. *International Seminar on Addressing Higher Order Thinking: Critical Thinking Issues in Primary Education*, *April*, 1–13.

Ayllon, M., Gomez, I., Ballesta-Claver, & Julio. (2016). Mathematical Thinking and Creativity Through Mathematical Problem Posing and Solving. *Propósitos y Representaciones*, *4*(1), 169–218. https://doi.org/http://dx.doi.org/10.20511/pyr2016.v4n1.89

Bailin, S., Case, R., Coombs, J. R., & Daniels, L. B. (2010). Conceptualizing Critical Thinking. *Journal of Curriculum Studies*, *31*(3), 285–302. https://doi.org/10.1080/002202799183133

Basri, H., Purwanto, As'ari, A. R., & Sisworo. (2019). Investigating Critical Thinking Skill of Junior High School in Solving Mathematical Problem. *International Journal of Instruction*, *12*(3), 745–758. https://doi.org/10.29333/iji.2019.12345a

Bassham, G., Irwin, W., Nardone, H., & Wallace, J. M. (2011). Critical Thinking: A Student Introduction. In *The McGraw-Hill Companies* (Fourth Edi). Mc Graw Hill. https://dimparato.files.wordpress.com/2016/08/critical-thinking.pd

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational-Number Concepts. In *R. Lesh & M. Landau (Eds.), Acquisition of Mathematics Concepts and Processes* (pp. 91–125).

Beyer, B. . (1987). Critical Thinking: What Is It? *Social Education*, *49*(4), 270-276. EJ316045

Bhaskar, R. (1975). *A Realistic Theory of Science: With a New Introduction*. Routledge Taylor & Francis Group. https://doi.org/10.2307/2215817

Binker, A. J. (1995). Strategies: Thirty-Five Dimensions of Critical Thinking. In *Critical thinking handbook: Foundation for Critical Thinking* (pp. 391–445). https://www.criticalthinking.org/data/pages/93/2dc1156cb915ed5e2d4de94d2bfe79e9513644fd4c683.pdf

Blitzer, R. F. (2019). *Thinking Mathematically* (7th ed.). Published by Pearson.

Breen, S., & O'Shea, A. (2010). Mathematical Thinking and Task Design. In *Irish Mathematical Society Bulletin* (Vol. 0066, Issue November 2010, pp. 39–49). https://doi.org/10.33232/bims.0066.39.49

Brousseau, G. (2002). Theory of Didactical Situations in Mathematics. In R. S. and V. W. Nicola Balacheff, Mantin Cooper (Ed.), *Kluwer Academic Publishers* (Edited and). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-

47211-2

Burrell, L. A. (2014). Integrating Critical Thinking Strategies into Nursing Curricula. *Teaching and Learning in Nursing*, *9*(2), 53–58. https://doi.org/10.1016/j.teln.2013.12.005

Burton, G. M. (1982). Patterning: Powerful Play. *School Science and Mathematics*, *82*(1), 39–44. https://doi.org/10.1111/j.1949-8594.1982.tb17161.x

Burton, L. (1984). Mathematical Thinking: The Struggle for Meaning. *Journal for Research in Mathematics Education*, *15*(1), 35–49. https://doi.org/10.5951/jresematheduc.15.1.0035

Butler, R. (2001). Book Reviews: Conceptual and Empirical Application. *Organization Studies*, *22*(2), 367–369. https://doi.org/10.1177/0170840601222008

Carpenter, T. P., Franke, M. L., Johnson, N. C., Turrou, A. C., & Wager, A. A. (2017). *Young Children's Mathematics: Cognitively Guided Instruction in Early Childhood Education*. Heinemann.

Chirove, M., Mogari, D., & Ugorji, O. (2022). Students' Mathematics-Related Belief Systems and Their Strategies for Solving Non-Routine Mathematical Problems. *Waikato Journal of Education*, *27*(3), 101–121. https://doi.org/10.15663/wje.v27i3.822

Choy, S. C., & Oo, P. S. (2012). Reflective Thinking and Teaching Practices: a Precursor for Incorporating Critical Thinking Into the Classroom? *International Journal of Instruction*, *5*(1), 167–182.

Clarke, D. M., & Roche, A. (2009). Students' Fraction Comparison Strategies as a Window into Robust Understanding and Possible Pointers for Instruction. *Educational Studies in Mathematics*, *72*(1), 127–138. https://doi.org/10.1007/s10649-009-9198-9

Costa, Arthur L., E. (1985). Developing Minds: A Resource Book for Teaching Thinking. In *Association for Supervision and Curriculum Development* (Revised Ed). https://doi.org/10.4324/9781315623511

Creswell, J. W. (2014). *Research Design Qualitative, Quantitatif, and Mixed Methods Approaches* (V. Knight (ed.); Fourth). SAGE Publications, Inc.

Critical thinking. Upper Saddle River: Prentice-Hall. (n.d.).

Department of Basic Education. (2011). Curriculum and Assessment Policy Statement: Further Education and Training Phase. Grades 10-12 Mathematical Literacy. In *Encyclopedia of Teacher Education*. https://doi.org/10.1007/978-981-16-8679-5_300016

Desmet, L., Grégoire, J., & Mussolin, C. (2010). Developmental changes in the

comparison of decimal fractions. *Learning and Instruction*, *20*(6), 521–532. https://doi.org/10.1016/j.learninstruc.2009.07.004

Dressel, P. L., & Mayhew, L. B. (1954). *General Education, Explorations in Education: The Final Report of the Cooperative Study of Evaluation in General Education of the American Council on Education.* American Council on Edu- cation,.

Emelia, E. (2017). Mengajar Berpikir Kritis dalam Menulis. *Jurnal Bahasa Dan Sastra FPBS UPI*, *7*(2).

Ennis, R. H. (1985). A Logica l Basis for Measuring Critical Thinking Skills. *Educational Leaderschip*, *oktober*, 44–48. https://pdfs.semanticscholar.org/80a7/c7d4a98987590751df4b1bd9adf747fd7aaa.pdf

Ennis, R. H. (1991). Critical Thinking: A Streamlined Conception. *Teaching Philosophy*, *14*(1), 5–24.

Ennis, R. H. (1996a). Critical Thinking. In *Upper Saddle River: Prentice-Hall.*

Ennis, R. H. (1996b). Critical Thinking Dispositions: Their Nature and Assessability. *Informal Logic*, 165–182.

Ennis, R. H. (2015). The Nature of Critical Thinking: Outlines of General Critical Thinking Disposition and Abilities. *Sixth International Conference on Thinking at MIT*, 1–8. https://education.illinois.edu/docs/default-source/faculty-documents/robert-ennis/thenatureofcriticalthinking

Ersoy, E., & Başer, N. (2014). The Effects of Problem-Based Learning Method in Higher Education on Creative Thinking. *Procedia - Social and Behavioral Sciences*, *116*(February 2014), 3494–3498. https://doi.org/10.1016/j.sbspro.2014.01.790

Facione, P. A. (1990). Critical Thinking: A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction. Research Findings and Recommendations. *American Philosophical Association*.

Facione, P. A. (2011). Critical Thinking : What It Is and Why It Counts. *Insight Assessment*, *ISBN 13: 978-1-891557-07-1.*, 1–28. https://www.insightassessment.com/CT-Resources/Teaching-For-and-About-Critical-Thinking/Critical-Thinking-What-It-Is-and-Why-It-Counts/Critical-Thinking-What-It-Is-and-Why-It-Counts-PDF

Facione, P. A. (2015). Critical Thinking : What It Is and Why It Counts. In *Insight assessment* (Issue ISBN 13: 978-1-891557-07-1.). https://www.insightassessment.com/

Facione, P. A. (2020). Critical Thinking: What It Is and Why It Counts. *Insight Development Program*, *53*(9), 1–31.

https://doi.org/10.1017/CBO9781107415324.004

Fadhilah, R., Sujadi, I., & Siswanto. (2021). The Critical Thinking Process of Senior High School Students in Problem-Solving of Linear Equations System. *IOP Conference Series: Earth and Environmental Science*, *1808*(1), 1–7. https://doi.org/10.1088/1742-6596/1808/1/012063

Fawcett, H. P. (1939). The Nature of Proof. A Description and Evaluation of Certain Procedures Used in Senior High School to Develop an understanding of the Nature of Proof. National Council of Teachers of Mathematics, Yearbook 13 (1938). Reprint 1966. In *Thought* (Vol. 14, Issue 3). https://doi.org/10.5840/thought1939143111

Fisher, A. (2009). *Berpikir Kritis: Sebuah Pengantar* (G. Sagara (Ed.)). Penerbit Erlangga.

Fisher, R. (1995). *Teaching Children to Think. Hong Kong: Stanley Thornes Ltd.* Stanley Thornes Ltd.

Foucault, M. (1984). *The Courage of the Truth* (F. Gros, F. E. Fontana, & Alessandro (Eds.)). Palgrave Macmillan.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing Children's Mathematical Thinking in Everyday Mathematics Classrooms. *Journal for Research in Mathematics Education*, *30*(2), 148–170. https://doi.org/10.2307/749608

Freudenthal, H. (1973). *Mathematics as An Educational Task. The Netherlands: Riedel Publishing Company*. Riedel Publishing Company.

Fusch, P., Fusch, G. E., & Ness, L. R. (2018). Denzin's Paradigm Shift: Revisiting Triangulation in Qualitative Research. *Journal of Social Change*, *10*(1). https://doi.org/10.5590/josc.2018.10.1.02

Gereti, L. C. V., & Savioli, A. M. P. das D. (2015). Advanced Mathematical Thinking Processes Evidenced in Resolutions of Questions of ENADE. *Bolema: Boletim de Educação Matemática*, *29*(51), 206–222. https://doi.org/10.1590/1980-4415v29n51a11

Ghanizadeh, A. (2017). The Interplay Between Reflective Thinking, Critical Thinking, Self-Monitoring, and Academic Achievement in Higher Education. *Higher Education*, *74*(1), 101–114. https://doi.org/10.1007/s10734-016-0031-y

Gokhale, A. A. (1995). Collaborative Learning Enhances Critical Thinking. *JTE*, *7*(1). https://doi.org/org/10.21061/jte.v7i1.a.2

Halpern, D. F. (2012). *Halpern Critical Thinking Assessment [Measurement instrument]. Mödling: Schuhfried*.

Halpern, D. F. (2014). Thought and Knowledge: An Introduction to Critical Thinking. In *Thought and Knowledge: An Introduction to Critical Thinking, Fifth Edition* (Fifth Edit). Psychology Press. https://doi.org/10.4324/9781315885278

Hamel, C., Turcotte, S., Laferrière, T., & Bisson, N. (2015). Improving Students ' Understanding and Explanation Skills Through the Use. *McGill Journal of Education*, *50*(1), 1–20.

Hammond, J., & Gibbons, P. (2005). Putting Scaffolding to Work: The Contribution of Scaffolding in Articulating ESL Education. *Prospect*, *20*(3), 6–30. https://doi.org/10.1016/0889-4906(95)90023-3

Harel, G. (2008). *What is Mathematics? A Pedagogical Answer to a Philosophical Question*. https://doi.org/10.5948/upo9781614445050.018

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *American Mathematical Society*, *7*, 234–283. https://doi.org/10.1090/cbmath/007/07

Hargreaves, M., Shorrocks-Taylor, D., & Threlfall, J. (1998). Children's Strategies with Number Patterns. *Educational Studies*, *24*(3), 315–331. https://doi.org/10.1080/0305569980240305

Henderson, P. B., Fritz, J., Hamer, J., Hitchner, L., Marion, B., Riedesel, C., & Scharff, C. (2002). Materials Development in Support of Mathematical Thinking. *Proceedings of the Conference on Integrating Technology into Computer Science Education, ITiCSE*, 185–190. https://doi.org/10.1145/960568.783001

Herman, T. (2000). Strategi Pemecahan Masalah (Problem Solving) dalam Pembelajaran Matematika. In *Makalah. Tidak Diterbitkan*. http://file.upi.edu/Direktori

Hitchcock, D. (2017). Critical Thinking as an Educational Ideal. In *On Reasoning and Argument, Argumentation Library* (pp. 477–497). © Springer International Publishing. https://doi.org/10.1007/978-3-319-53562-3_30

Hitchcock, D. (2018). Critical Thinking. In *Stanford Encyclopedia of Philosophy* (First publ). Stanford Encyclopedia of Philosophy.

Inch, E., & Tudor, K. (2006). *Critical Thinking and Communication: The Use of Reason in Argument* (7th Editio). Pearson Education, Inc.

Jenicek, M., Croskerry, P., & Hitchcock, D. (2011). Evidence and Its Uses in Health Care and Research: The Role of Critical Thinking. *Medical Science Monitor*, *17*(1), 12–17. https://doi.org/10.12659/msm.881321

Jonassen, D. H. (1997). Instructional Design Models for Well-Structured and Ill-Structured Problem-Solving Learning Outcomes. *ETR & D*, *45*(1), 24th Feb.

https://doi.org/10.1007/BF02299613

Jones, B. F., Palinesar, A. S., Ogle, D. S., & Carr, E. G. (1987). *Strategic Teaching and Learning: Cognitive Instruction in the Content Areas.* ED286858.pdf

Kamus Online. (2019). *Wikipedia*. http wikipediafoundation.org/fundraising

Kattsoff, L. (1996). *Element of Philosophy. Terjemahan Soemargono. Pengantar Filsafat*. Tiara acana.

Kenneth, R. (1986). *Teaching the Process of Thinking, K-12* (D. L. Burleson (Ed.)). Delta Kappa Educational Foundation.

Keskin, M., Akbaba, S., & Altun, M. (2013). Comparison of 8th and 11th grade Students Behaviours at Mathematical Thinking. *Journal of Educational Sciences*, *33*(1), 33–50.

Klein, P. S., Adi-Japha, E., & Hakak-Benizri, S. (2010). Mathematical Thinking of Kindergarten Boys and Girls: Similar Achievement, Different Contributing Processes. *Educational Studies in Mathematics*, *73*(3), 233–246. https://doi.org/10.1007/s10649-009-9216-y

Knuth, E. J. (2002). Proof as a Tool for Learning Mathematics. *The Mathematics Teacher*, *95*(7), 486–490. https://doi.org/10.5951/mt.95.7.0486

Krammer, K. (2009). *Individuelle Lernunterstützung in Schülerarbeitsphase. Eine videobasierte Analyse des Unterstützungsverhaltens von Lehrpersonen im Mathe- matikunterricht. Münster: Waxmann.*

Krulik, S., & Rudnick, J. A. (1988). *Problem Solving: A Handbook for Elemen School Teachers*. Allyn and Bacon, Inc.

Kuntze, S., Aizikovitsh, E., & David, U. (2017). Hybrid task design : connecting learning opportunities related to critical thinking and statistical thinking. *ZDM*, *0*(0), 0. https://doi.org/10.1007/s11858-017-0874-4

Lai, E. R. (2011). Critical Thinking: A Literature Review. Research Report. *Pearson*, *35*(3), 219–225. https://doi.org/10.1046/j.1537-2995.1995.35395184278.x

Lane, C. P., & Harkness, S. S. (2012). Game Show Mathematics: Specializing, Conjecturing, Generalizing, and Convincing. *Journal of Mathematical Behavior*, *31*(2), 163–173. https://doi.org/10.1016/j.jmathb.2011.12.008

Lee, K., Ng, S. F., Bull, R., Lee Pe, M., & Ho, R. H. M. (2011). Are Patterns Important? An Investigation of the Relationships Between Proficiencies in Patterns, Computation, Executive Functioning, and Algebraic Word Problems. *Journal of Educational Psychology*, *103*(2), 269–281. https://doi.org/10.1037/a0023068

Lepper, Drake, & O'Donnell-Johnson, T. (1997). Scaffolding techniques of expert human tutors. In *K. Hogan & M. Pressley (Eds.), Scaffolding student learning: Instructional approaches and issues* (pp. 108–144).

Liljedahl, P. (2004). Repeating Pattern or Number Pattern: The Distinction is Blurred. *Focus on Learning Problems in Mathematics*, *26*(3), 24–42. https://peterliljedahl.com/wp-content/uploads/JA-FLPM-2004.pdf

Liu, P. (2003). Do Teachers Need to Incorporate the History of Mathematics in Their Teaching? *Mathematics Teacher*, *96*(6), 416–421.

Lortie-Forgues, H., & Siegler, R. S. (2017). Conceptual Knowledge of Fraction Arithmetic. *Journal of Educational Psychology*, *209*(3), 374–386.

Marpaung, Y. (2006). *Psikologi Kognitif, Hand Out Perkuliahan. UNESA Surabaya*.

Marzano, R. J., Brandt, R. S., Hughes, C. S., Jones, B. F., Presseisen, B. Z., Rankin, S. C., & Suhor, C. (1988). Dimensions of Thinking: A Framework for Curriculum and Instruction. In *Journal of Petrology* (Vol. 369, Issue 1). https://doi.org/10.1017/CBO9781107415324.004

Mason, J., Burton, L., & Stacey, K. (2010). Thinking Mathematically. In *Pearson* (Second Edi). https://doi.org/10.12968/eyed.2013.15.2.18

Mayer, R. E. (2010). Problem Solving and Reasoning. *International Encyclopedia of Education*, 273–278. https://doi.org/10.1016/B978-0-08-044894-7.00487-5

Moleong, L. J. (2017). *Metode Penelitian Kualitatif* (Revisi). Remaja Rosdakarya.

Montague, M., Enders, C., & Dietz, S. (2011). Effects of Cognitive Strategy instruction on Math Problem Solving of Middle School With Learning Disabilities. *Learning Disability Quarterly*, *34*(4), 262–272.

Muhtadi, D., Supratman, & Hermanto, R. (2019). The students' mathematical critical thinking process reviewed from the cognitive style. *Journal of Physics: Conference Series*, *1188*(1). https://doi.org/10.1088/1742-6596/1188/1/012082

Murdiyani, N. M. (2018). Developing Non-Routine Problems for Assessing Students' Mathematical Literacy. *Journal of Physics: Conference Series*, *983*(1). https://doi.org/10.1088/1742-6596/983/1/012115

National Council of Teachers of Mathematics (NCTM). (2000). Executive Summary Principles and Standards for School Mathematics. *National Council of Teachers of Mathematics, Inc.*, 1–6.

NCTM. (2000). Principles Standards and for School Mathematics. In *National Council of Teachers of Mathematics* (Vol. 7, Issue 1, pp. 1–419).

Nickels, M., & Cullen, C. J. (2017). Mathematical thinking and learning through robotics play for children with critical illness: The case of Amelia. In *Journal for Research in Mathematics Education* (Vol. 48, Issue 1). https://doi.org/10.5951/jresematheduc.48.1.0022

Norman, M., Chang, P., & Prieto, L. (2017). Stimulating Critical Thinking in U.S Business Students through the Inclusion of International Students. *The Journal of Business Diversity*, *17*(1), 122–130.

Nugroho, P. B., Nusantara, T., As'ari, A. R., Sisworo, Hidayanto, E., & Susiswo. (2018). Critical Thinking Disposition: Students Skeptic in Dealing With Ill-Logical Mathematics Problem. *International Journal of Instruction*, *11*(3), 635–648. https://doi.org/10.12973/iji.2018.11343a

OECD. (2019). PISA 2018 Results. Combined Executive Summaries. *Journal of Chemical Information and Modeling*, *53*(9), 1689–1699.

Omar, N., Haris, S. S., Hassan, R., Arshad, H., Rahmat, M., Zainal, N. F. A., & Zulkifli, R. (2012). Automated Analysis of Exam Questions According to Bloom's Taxonomy. *Procedia - Social and Behavioral Sciences*, *59*(October), 297–303. https://doi.org/10.1016/j.sbspro.2012.09.278

Paul, R. W., A. Binker, K. J. H. K. (1990). Critical Thinking Handbook: 4. – 6. Grades A Guide For Remodelling Lesson Plans In Language Arts, Social Studies and Science. Rohnert Park, Ca, Foundation For Critical Thinking. Shania State University. In *Foundation for Critical Thinking Sonoma State University*.

Paul, R., & Elder, L. (2006). Critical Thinking: Concepts and Tools. In *Performance + Instruction* (Vol. 34, Issue 6). Foundation for Critical Thinking. https://doi.org/10.1002/pfi.4170340606

Perkins, C., & Murphy, E. (2006). Identifying and Measuring Individual Engagement in Critical Thinking in Online Discussions: An Exploratory Case Study. *Educational Technology and Society*, *9*(1), 298–307.

Pfister, M., Moser Opitz, E., & Pauli, C. (2015). Scaffolding for Mathematics Teaching in Inclusive Primary Classrooms: A Video Study. *ZDM - Mathematics Education*, *47*, 1079–1092. https://doi.org/10.1007/s11858-015-0713-4

Polya, G. (1985). How to Solve It. In *Princeton University Press* (Second Edi).

Pratiwi, I. (2019). Efek Program Pisa Terhadap Kurikulum Di Indonesia. *Jurnal Pendidikan Dan Kebudayaan*, *4*(1), 51. https://doi.org/10.24832/jpnk.v4i1.1157

Prediger, S. (2008). The Relevance of Didactic Categories for Analysing Obstacles in Conceptual Change: Revisiting the Case of Multiplication of Fractions. *Learning and Instruction*, *18*(1), 3–17.

https://doi.org/10.1016/j.learninstruc.2006.08.001

Rasiman. (2013). *Proses Berpikir Kritis Siswa dalam Menyelesaikan Masalah Matematika Ditinjau dari Perbedaan Kemampuan Matematika*. Unesa.

Ricoeur, P. (1986). *Lectures on Ideology and Utopia*. New York: Columbia University Press. https://archive.org/details/pdfy-oRPzWEh3nXrYxehT/page/n13/mode/2up

Rokhmat, J., Marzuki, Wahyudi, & Putrie, S. D. (2019). A Strategy of Scaffolding Development to Increase Students' Problem-Solving Abilities: The Case of Physics Learning with Causalitic-Thinking approach. *Journal of Turkish Science Education*, *16*(4), 569–579. https://doi.org/10.36681/tused.2020.8

Rønning, F. (2021). Opportunities for Language Enhancement in a Learning Environment Designed on the Basis of the Theory of Didactical Situations. *ZDM - Mathematics Education*, *53*(2), 305–316. https://doi.org/10.1007/s11858-020-01199-x

Ruggiero, & Vincent, R. (1998). *The Art of Thinking. A Guide to Critical and Creative Throught.* Longman An Imprit of Addison Wesley Longman, Inc.

Schoenfeld, A. H. (1992). Learning To Think Mathematically: Problem Solving, Metacognition, And Sense-Making In Mathematics. In *Handbook for Research on Mathematics Teaching and Learning*.

Schoenfeld, A. H. (2016). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Reprint). *Journal of Education*, *196*(2), 1–38. https://doi.org/10.1177/002205741619600202

Scusa, T., & Yuma. (2008). Five Processes of Mathematical Thinking. *DigitalCommons@University of Nebraska - Lincoln*, *7*, 1–60. https://digitalcommons.unl.edu/mathmidsummative/38

Setiana, D. S., Purwoko, R. Y., & Sugiman. (2021). The application of mathematics learning model to stimulate mathematical critical thinking skills of senior high school students. *European Journal of Educational Research*, *10*(1), 509–523. https://doi.org/10.12973/EU-JER.10.1.509

Seventika, S. Y., Sukestiyarno, Y. L., & Mariani, S. (2018). Critical Thinking Analysis Based on Facione (2015) - Angelo (1995) Logical Mathematics Material of Vocational High School (VHS). *Journal of Physics: Conference Series*, *983*(1). https://doi.org/10.1088/1742-6596/983/1/012067

Shawan, M., Osman, S., & Abu, M. S. (2021). Difficulties in Solving Non-Routine Problems: Preliminary Analysis and Results. *ASM Science Journal*, *16*(3), 1–14. https://doi.org/10.32802/asmscj.2021.800

Shehab, H. M., & Nussbaum, E. M. (2015). Cognitive Load of Critical Thinking Strategies. *Learning and Instruction*, *35*, 51–61.

https://doi.org/10.1016/j.learninstruc.2014.09.004

Simpson, E., & Courtney, M. (2008). Implementation and Evaluation of Critical Thinking Strategies to Enhance Critical Thinking Skills in Middle Eastern Nurses. *International Journal of Nursing Practice*, *14*(6), 449–454. https://doi.org/10.1111/j.1440-172X.2008.00719.x

Siswono, T. Y. E. (2007). *Penjenjangan Kemampuan Berpikir Kreatif dan Identifikasi Tahap Berpikir Kreatif Siswa Dalam Memecahkan dan Mengajukan Masalah Matematika.* Unesa Surabaya.

Siswono, T. Y. E. (2016). Proses Berpikir Kreatif Siswa Dalam Memecahkan dan Mengajukan Masalah Matematika. *Jurnal Ilmu Pendidikan*, *15*(1), 1–14. https://doi.org/10.17977/jip.v15i1.13

Smith, J. A., & Eatough, V. (2019). Looking Forward: Conceptual and Methodological Developments in Interpretative Phenomenological Analysis: Introduction to the Special Issue. *Qualitative Research in Psychology*, *16*(2), 163–165. https://doi.org/10.1080/14780887.2018.1540620

Solso, R. L., MacLin, O. H., & MacLin, M. K. (2008). *Cognitive psychology (8th Edition)*. Pearson Education.

Sönmez, E., Kabataş Memiş, E., & Yerlikaya, Z. (2019). The Effect of Practices Based on Argumentation-Based Inquiry Approach on Teacher Candidates' Critical Thinking. *Educational Studies*, *47*(1), 59–83. https://doi.org/10.1080/03055698.2019.1654364

Stacey, K. (2006). What Is Mathematical Thinking and Why Is It Important? *Review of Educational Research*, *82*(3), 330–348.

Stafylidou, S., & Vosniadou, S. (2004). The Development of Students' Understanding of the Numerical Value of Fractions. *Learning and Instruction*, *14*(5 SPEC.ISS.), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015

Sumarmo, U. (2010). Berfikir dan Disposisi Matematik: Apa, Mengapa, dan Bagaimana Dikembangkan Pada Peserta Didik. *Fpmipa Upi*, 1–27.

Suryadi, D. (2011a). Membangun Budaya Baru dalam Berpikir Matematika. In *SPs UPI*. https://doi.org/10.1017/CBO9781107415324.004

Suryadi, D. (2011b). Metapedadidaktik dalam Pembelajaran Matematika : Suatu Strategi Pengembangan Diri Menuju Guru Matematika Profesional [Metapedadidactics in Mathematics Learning: A Self-Development Strategy Towards Professional Mathematics Teachers]. Pidato Pengukuhan Guru. In *Tidak Diterbitkan*.

Suryadi, D. (2013). Didactical Design Research (DDR) Dalam Pengembangan Pembelajaran Matematika. *Prosiding Seminar Nasional Matematika Dan Pendidikan Matematika STKIP Siliwangi Bandung*, *1*, 3–12.

Suryadi, D. (2019a). *Landasan Filosofis Penelitian Desain Didaktis (DDR) [Philosophical Foundations of Didactic Design Research (DDR)]*. (T. G. Press (Ed.); Cetakan 1). Gapura Press.

Suryadi, D. (2019b). *Penelitian Desain Didaktical (DDR) dan Implementasinya* (A. S. Maulida (Ed.); Cetakan 1). Gapura Press.

Suryadi, D., Yulianti, K., & Junaeti, E. (2005). *Model Antisipasi dan Situasi Didaktis* (pp. 1–10).

Sutini, S., Sutawidjaja, A., Parta, I. N., & Susanto, H. (2017). Identification of Critical Thinking Process in Solving Mathematic Problems. *IOSR Journal of Research & Method in Education*, *07*(04), 05–10.

Tall, D. (2002a). Advanced Mathematical Thinking. In *Kluwer Academic Publishers*.

Tall, D. (2002b). The Psychology of Advanced Mathematical Thinking. In *Advanced Mathematical Thinking* (Vol. 21, pp. 3–21). https://doi.org/10.1007/0-306-47203-1_1

Threlfall, J. (1999). Repeating Patterns in the Early Primary Years. In *A. Orton (Ed.), Patterns in the teaching and learning of mathematics* (pp. 18–30).

Tohir, M., Maswar, M., Moh, A., Saiful, S., Pradita, R., & Ayu, D. (2020). Prospective Teachers' Epectations of Students' Mathematical Thinking Processes in Solving Problems. *European Journal of Educational Research*, *9*(4), 1735–1748. https://doi.org/10.12973/EU-JER.9.4.1735

Trilling and Fadel. (2009). 21st Century Skills: Learning for Life in Our Times. Jossey Bass: USA. *Journal of Chemical Information and Modeling*, *53*(9), 1689–1699.

Unwakoly, S. (2022). Berpikir Kritis Dalam Filsafat Ilmu: Kajian Dalam Ontologi, Epistemologi dan Aksiologi. *Jurnal Filsafat Indonesia*, *5*(2), 95–102. https://ejournal.undiksha.ac.id/index.php/JFI/article/view/42561

Uyangör, S. M. (2019). Investigation of the Mathematical Thinking Processes of Students in Mathematics Education Supported with Graph Theory. *Universal Journal of Educational Research*, *7*(1), 1–9. https://doi.org/10.13189/ujer.2019.070101

Valentová, M., & Brečka, P. (2019). Implementation of the critical thinking strategies in the school subject technology: A preliminary study. *TEM Journal*, *8*(3), 998–1004. https://doi.org/10.18421/TEM83-43

Vamvakoussi, X., Christou, K. P., Mertens, L., & Van Dooren, W. (2011). What fills the gap between discrete and dense? Greek and Flemish students' understanding of density. *Learning and Instruction*, *21*(2011), 676–685. https://doi.org/10.1016/j.learninstruc.2011.03.005

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction*, *14*(5 SPEC.ISS.), 453–467. https://doi.org/10.1016/j.learninstruc.2004.06.013

Vamvakoussi, X., & Vosniadou, S. (2010). How many decimals are there between two fractions? Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition & Instruction*, *28*. https://doi.org/10.1080/07370001003676603

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: A decade of research. *Educational Psychology Review*, *22*(3), 271–296. https://doi.org/10.1007/s10648-010-9127-6

Van Der Stuyf, R. R. (2002). Scaffolding as a Teaching Strategy – Definition and Description. In *Adolescent Learning and Development* (pp. 1–13).

Vygotsky, L. S. (1978). Mind in Society: The Development ofHigher Psychological Processes. In *Cambridge Massachusetts*. Harvard University Press. https://doi.org/10.3928/0048-5713-19850401-09

Wass, R., Harland, T., & Mercer, A. (2011). Scaffolding critical thinking in the zone of proximal development. *Higher Education Research and Development*, *30*(3), 317–328. https://doi.org/10.1080/07294360.2010.489237

Watson, G., & Glaser, E. (2002). Watson Glaser Critical Thinking Appraisal Test. In *Pearson Assessment*.

Williams, R. (1974). Communications As Cultural Science. *Journal of Communication*, *24*(3), 17–25. https://doi.org/10.1111/j.1460-2466.1974.tb00385.x

Wilson, K. (2016). Critical Reading, Critical Thinking: Delicate Scaffolding in English for Academic Purposes (EAP). *Thinking Skills and Creativity*, *22*, 256–265. https://doi.org/10.1016/j.tsc.2016.10.002

Wood, D., Bruner, J. S., & Ross, G. (1976). The Role of Tutoring in Problem Solving. *Journal of Child Psychology and Psychiatry*, *17*(2), 89–100.

Yıldırım, D., & Köse, N. Y. (2018). Mathematical Thinking Processes of Secondary School Students in Polygon Problems. Abant İzzet Baysal University, , 18 (1), 605-633, 2018. *Education Faculty Journal*, *88*(1), 605–633.