

**ANALISIS PROSES BERPIKIR GEOMETRI  
MAHASISWA CALON GURU MATEMATIKA DALAM PERSPEKTIF  
*WAY OF THINKING* DAN *WAY OF UNDERSTANDING*  
PADA IMPLEMENTASI DESAIN DIDAKTIS  
BERBANTUAN *SOFTWARE* GEOGEBRA**

**Disertasi**

*Diajukan untuk Memenuhi Sebagian dari Persyaratan Memperoleh Gelar  
Doktor Pendidikan Matematika*



Promovendus:  
Thesa Kandaga  
NIM. 1707729

**PROGRAM STUDI PENDIDIKAN MATEMATIKA  
FAKULTAS PENDIDIKAN MATEMATIKA DAN ILMU PENGETAHUAN ALAM  
UNIVERSITAS PENDIDIKAN INDONESIA  
2022**

**ANALISIS PROSES BERPIKIR GEOMETRI MAHASISWA CALON GURU MATEMATIKA  
DALAM PERSPEKTIF *WAY OF THINKING* DAN *WAY OF UNDERSTANDING*  
PADA IMPLEMENTASI DESAIN DIDAKTIS INTEGRASI *DYNAMIC GEOMETRY SOFTWARE***

Oleh

Thesa Kandaga

Dr. Universitas Pendidikan Indonesia, 2022  
M. Pd. Universitas Pendidikan Indonesia, 2013

Sebuah Disertasi yang diajukan untuk memenuhi salah satu syarat memperoleh gelar Doktor Pendidikan (Dr.) pada Fakultas Pendidikan Matematika dan Ilmu Pengetahuan Alam

© Thesa Kandaga 2022  
Universitas Pendidikan Indonesia  
Juli 2022

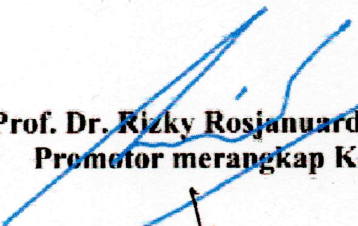
Hak Cipta dilindungi undang-undang.  
Disertasi ini tidak boleh diperbanyak seluruhnya atau sebagian, dengan dicetak ulang, difoto kopi, atau cara lainnya tanpa ijin dari penulis.

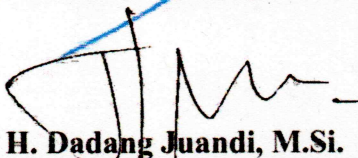
**HALAMAN PENGESAHAN**

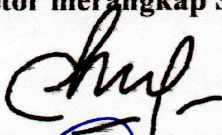
**THESA KANDAGA**

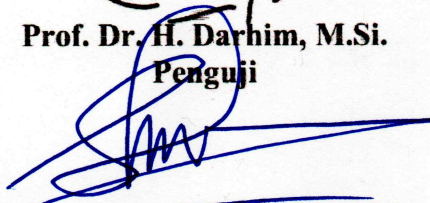
**ANALISIS PROSES BERPIKIR GEOMETRI  
MAHASISWA CALON GURU MATEMATIKA DALAM PERSPEKTIF  
*WAY OF THINKING* DAN *WAY OF UNDERSTANDING*  
PADA IMPLEMENTASI DESAIN DIDAKTIS  
BERBANTUAN *DYNAMIC GEOMETRY SOFTWARE***

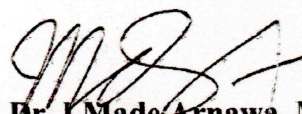
Disetujui dan Disahkan Oleh Panitia Disertasi  
untuk Diajukan pada Ujian Tahap II

  
**Prof. Dr. Rizky Rosjanuardi, M.Si.**  
Promotor merangkap Ketua

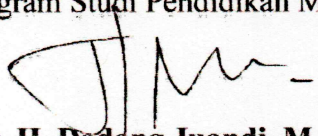
  
**Dr. H. Dadang Juandi, M.Si.**  
Co-promotor merangkap Sekertaris

  
**Prof. Dr. H. Darhim, M.Si.**  
Penguji

  
**Siti Fatimah, S.Pd., M.Si., Ph.D.**  
Penguji

  
**Prof. Dr. I Made Arnawa, M.Si.**  
Penguji Luar Universitas

Mengetahui,  
Ketua Program Studi Pendidikan Matematika

  
**Dr. H. Dadang Juandi, M.Si.**  
NIP. 196401171992021001

## ABSTRAK

### **Thesa Kandaga (1707729). Analisis Proses Berpikir Geometri Mahasiswa Calon Guru Matematika Dalam Perspektif *Way of Thinking* dan *Way of Understanding* Pada Implementasi Desain Didaktis Berbantuan *Software GeoGebra***

Pembuatan sketsa atau gambar menggunakan pensil dan kertas menjadi kontraproduktif dalam pembelajaran daring. Oleh karena itu, diperlukan desain didaktis untuk mengintegrasikan penggunaan *Dynamic Geometry Software* (DGS) ke dalam pembelajaran, agar dapat menunjang proses berpikir mahasiswa sesuai dengan Teori van Hiele. Kemudian dipilih sepuluh orang untuk menjadi subjek penelitian. Data yang dikumpulkan dari subjek kemudian dianalisis secara kualitatif untuk mendeskripsikan proses berpikir geometri subjek dalam model van Hiele dengan tujuan sebagai berikut: (1) mengidentifikasi hambatan belajar dalam beberapa topik geometri transformasi, (2) merancang desain didaktis hipotetis untuk pembelajaran geometri transformasi berbantuan DGS, (3) mendeskripsikan hasil penerapan desain didaktis untuk pembelajaran geometri transformasi berbantuan DGS dalam Teori Van Hiele, (4) mendeskripsikan proses berpikir geometri mahasiswa calon guru menurut model van Hiele dalam perspektif *way of thinking*, (5) mendeskripsikan proses berpikir geometri mahasiswa calon guru menurut model van Hiele dalam perspektif *way of understanding*, dan (6) merancang desain didaktis empiris untuk pembelajaran geometri transformasi berbantuan DGS berdasarkan analisis retrospektif. Berdasarkan hasil penelitian, didapatkan kesimpulan: (1) terdapat hambatan epistemologis dalam setiap level berpikir model van Hiele pada topik geometri transformasi, (2) terdapat efek samping dari penggunaan DGS dalam pembelajaran, (3) munculnya *way of thinking* baru dari implementasi desain didaktis, dan (4) perspektif *way of thinking* dan *way of understanding* dapat menjelaskan kedinamisan dan kontinuitas setiap level van Hiele.

**Kata Kunci:** Desain didaktis, Geometri Transformasi, Teori van Hiele, *Way of Thinking*, *Way of Understanding*, *Dynamic Geometry Software*

## ABSTRACT

**Thesa Kandaga (1707729). Analysis of Prospective Mathematics Teacher's Geometric Thinking in The Way of Thinking and Way of Understanding Perspective on The Implementation of Didactical Design Assisted by Dynamic Geometry Software.**

Sketching or drawing using pencil and paper is becoming counterproductive in online learning. Therefore, in order to support students' geometric thinking in accordance to van Hiele's Theory, didactic design is needed to integrate Dynamic Geometry Software (DGS) into learning. Ten people were selected to be research subjects. The data collected was then analyzed qualitatively to describe the subject's geometric thinking accordance to van Hiele's model with the following objectives: (1) identify learning obstacles in geometric transformation topics, (2) designing hypothetical didactic design for geometric transformation topics assisted by DGS, (3) describe the results of implementing didactic design for geometric transformation according to van Hiele's Theory, (4) describe prospective teacher's geometric thinking according to the van Hiele model in ways of thinking perspective, (5) describe prospective teacher's geometric thinking according to the van Hiele model in way of understanding perspective, and (6) designing empirical didactic design for geometric transformation topics assisted by DGS based on the retrospective analysis. Based on the results, concluded that: (1) There are epistemological obstacles in each level of geometric transformation topics accordance to van Hiele's model, (2) there are side effects from using DGS in learning, (3) emergence of new way of thinking from the didactic design, and (4) way of thinking and way of understanding perspectives can explain the dynamics and continuity at each van Hiele's level.

**Keywords:** Didactical design, Geometric Transformation, van Hiele Theory, Way of Thinking, Way of Understanding, Dynamic Geometry Software

## DAFTAR ISI

KATA PENGANTAR .....	i
DAFTAR ISI.....	iii
DAFTAR TABEL.....	vii
DAFTAR GAMBAR.....	x
DAFTAR LAMPIRAN.....	xv
ABSTRAK.....	xvi
ABSTRACT.....	xvii
BAB I PENDAHULUAN.....	1
1.1 Latar Belakang .....	1
1.2 Rumusan Masalah .....	13
1.3 Tujuan Penelitian.....	14
1.4 Manfaat Penelitian.....	14
1.5 Definisi Operasional.....	15
BAB II LANDASAN TEORITIS .....	17
2.1 <i>Didactical Design Research</i> .....	17
2.2 Teori Level Berpikir Geometri Model van Hiele .....	25
2.2.1 Level dari Teori Berpikir Geometri Model Van Hiele .....	26
2.2.1.1 Level-1 (Visualisasi).....	30
2.2.1.2 Level-2 (Analisis) .....	33
2.2.1.3 Level-3 (Abstraksi) .....	34
2.2.1.4 Level-4 (Deduksi) .....	35
2.2.1.5 Level-5 ( <i>Rigor</i> ) .....	37
2.2.2 Sifat-sifat Tiap Level .....	41
2.2.3 Implementasi dalam Mata Kuliah Geometri Transformasi .....	42
2.2.3 Perkembangan dari Suatu Level ke Level Selanjutnya .....	42
2.2.4 Tahapan Pembelajaran dalam Teori Van Hiele .....	43
2.2.4.1 <i>Information</i> .....	43
2.2.4.2 <i>Directed orientation</i> .....	44
2.2.4.3 <i>Explicitation</i> .....	44
2.2.4.4 <i>Free orientation</i> .....	45

2.2.4.5 <i>Integration</i> .....	45
2.2.5 Karakteristik Umum Teori Van Hiele .....	46
2.3 <i>Way of Understanding</i> dan <i>Way of Thinking</i> .....	46
2.4 <i>Dynamic Geometry Software</i> .....	52
2.4.1 GeoGebra untuk Demonstrasi, Simulasi dan Visualisasi .....	56
2.4.2 GeoGebra Sebagai Alat Bantu Konstruksi Konsep .....	57
2.4.3 GeoGebra untuk Eksplorasi dan Penemuan Matematika .....	58
2.4.4 GeoGebra Sebagai Perangkat Lunak Pembangun Bahan Ajar ( <i>Authoring Tools</i> ).....	60
2.4.5 GeoGebra untuk Menyelesaikan atau Memverifikasi Permasalahan Matematika.....	61
2.5 Teori Belajar Lain yang Mendukung .....	61
2.5.1 <i>Theory of Didactical Situation</i> .....	61
2.5.1.1 Aksi.....	63
2.5.1.2 Formulasi .....	64
2.5.1.3 Validasi .....	64
2.5.1.4 Institusionalisasi.....	65
2.5.2 Teori Vygotsky .....	72
2.5.2.1 Zona Pengembangan Proximal .....	74
2.5.2.2 Teknik <i>Scaffolding</i> .....	75
2.6 Hasil Penelitian Lain yang Relevan .....	78
BAB III METODOLOGI PENELITIAN .....	83
3.1 Pendekatan dan Metode Penelitian.....	83
3.1.1 Tahap Penyusunan Antisipasi Didaktis-Pedagogis (Desain Didaktis Hipotesis).....	84
3.1.2 Tahap Implementasi dan Metapedadidaktik.....	85
3.1.3 Tahap Analisis Retrospektif .....	86
3.2 Metode Penelitian.....	87
3.3 Subjek Penelitian.....	88
3.4 Tempat dan Waktu Penelitian .....	90
3.5 Teknik Pengumpulan Data .....	91
3.6 Teknik Analisis Data .....	92

3.7 Kriteria Keabsahan Data .....	96
3.7.1 Kriteria <i>Credibility</i> .....	96
3.7.1.1 Menggunakan Koleksi Bahan Referensi yang Memadai.....	96
3.7.1.2 Diskusi dengan Ahli/Peneliti Lain .....	96
3.7.1.3 Triangulasi .....	97
3.7.1.4 <i>Member Check</i> .....	97
3.7.2 Kriteria <i>Transferability</i> .....	97
3.7.3 Kriteria <i>Dependability</i> .....	98
3.7.4 Kriteria <i>Confirmability</i> .....	99
<b>BAB IV HASIL PENELITIAN DAN PEMBAHASAN.....</b>	<b>102</b>
4.1 Analisis Hambatan Belajar.....	102
4.1.1 Hambatan Ontogenik.....	102
4.1.2 Hambatan Didaktis .....	109
4.1.3 Hambatan Epistemologis .....	114
4.1.3.1 Hambatan Epistemologis pada Level Visualisasi .....	120
4.1.3.2 Hambatan Epistemologis pada Level Analisis .....	122
4.1.3.3 Hambatan Epistemologis pada Level Abstraksi .....	124
4.1.3.4 Hambatan Epistemologis pada Level Deduksi .....	127
4.1.3.5 Hambatan Epistemologis pada Level <i>Rigor</i> .....	130
4.2 Repersonalisasi dan Rekontekstualisasi .....	133
4.3 Desain Didaktis Hipotetik .....	140
4.4 Pembelajaran Daring .....	142
4.5 Peran <i>Dynamic Geometry Software</i> dalam Pembelajaran Geometri Transformasi .....	143
4.6 Fase Pembelajaran van Hiele .....	145
4.7 Pengembangan Desain Didaktis.....	146
4.7.1 <i>Lesson Design I</i> .....	148
4.7.2 <i>Lesson Design II</i> .....	153
4.7.3 <i>Lesson Design III</i> .....	161
4.7.4 <i>Lesson Design IV</i> .....	168
4.7.5 <i>Lesson Design V</i> .....	175
4.7.6 <i>Lesson Design VI</i> .....	179



4.8 Analisis Metapedadidaktik.....	182
4.8.1 Hasil Implementasi Desain Didaktis Pertemuan 1 .....	182
4.8.2 Hasil Implementasi Desain Didaktis Pertemuan 2 .....	184
4.8.3 Hasil Implementasi Desain Didaktis Pertemuan 3 .....	185
4.8.4 Hasil Implementasi Desain Didaktis Pertemuan 4 .....	187
4.8.5 Hasil Implementasi Desain Didaktis Pertemuan 5 .....	190
4.8.6 Hasil Implementasi Desain Didaktis Pertemuan 6 .....	192
4.9 Analisis Retrospektif.....	192
4.9.1 Temuan dan Analisis pada Situasi Didaktis .....	192
4.9.2 Temuan dan Analisis Hambatan Belajar .....	198
4.9.3 Analisis <i>Way of Thinking</i> dan <i>Way of Understanding</i> Partisipan....	209
4.9.3.1 Visualisasi Sebagai Dukungan Terhadap <i>Way of Thinking</i> .....	213
4.9.3.2 Desain Didaktis untuk <i>Way of Thinking</i> Deduksi dan Induksi .....	217
4.9.4 Efek Samping Penggunaan GeoGebra pada <i>Way of Thinking</i> dan <i>Way of Understanding</i> Partisipan.....	219
4.9.5 Penjelasan Kedinamisan dalam Proses Berpikir Geometri Van Hiele.....	223
4.9.6 Desain Didaktis Empiris.....	227
4.10 Keterbatasan Penelitian .....	235
BAB V KESIMPULAN DAN SARAN .....	237
5.1 Kesimpulan.....	237
5.2 Temuan Penting Dalam Penelitian .....	238
5.2 Implikasi dan Rekomendasi .....	239
DAFTAR PUSTAKA.....	241
LAMPIRAN .....	255

## DAFTAR PUSTAKA

- Adelabu, F.M, Makgato, M., & Ramaligela, M.S., (2019). The Importance of Dynamic Geometry Computer Software on Learners' Performance in Geometry. *The Electronic Journal of e-Learning*, 17(1), 52-63.
- Akasah, Z. A., & Alias, M. (2010). Bridging The Spatial Visualisation Skills Gap Through Engineering Drawing Using The Whole-to-Parts Approach. *Australasian Journal of Engineering Education*, 16(1), 81-86
- Alex, J. K., & Mammen, K. J. (2016). Lessons Learnt from Employing van Hiele Theory Based Instruction in Senior Secondary School Geometry Classrooms. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(8), 2223–2236. doi: 10.12973/eurasia.2016.12228a
- Anderson, G., Buck, D., Coates, T., & Corti, A. (2015). Drawing in Mathematics: From Inverse Vision to The Liberation of Form. *Leonardo*. 48(5), 439–448. doi: 10.1162/LEON\_a\_00909
- Arias, F. & Araya, A. (2009). Analysis of The Didactical Contracts in 10th Grade Math Classes. *Quaderni di Ricerca in Didattica (Matematica)*, 4(19), 155-163
- Ausubel, D.P. (1968). *Educational Psychology: A Cognitive View*. Holt, Rinehart and Winston: New York.
- Azizul, S. M. J., & Din, R. (2016). Teaching and Learning Geometry Using Geogebra Software via Mooc. *Journal of Personalized Learning*, 2(6), 39-50.
- Bell, F.H. (1978). *Teaching and Learning Mathematics (in Secondary Schools)*. Dubuque, Iowa: WM. C. Brown Company Publisher.
- BK-DPR-RI. (2021). *Reformasi Pendidikan: Pembelajaran Digital di Indonesia*. Analisis RUU Tentang APBN. Jakarta: DPR-RI.
- Barquero, E., Bosch, M. & Gascón, J. (2007). Los recorridos de estudio e investigación en la reformulación didáctica del problema de la metacognición. *Sociedad, Escuela y Matemáticas*, 481-50

- Chang, C-Y & Bhagat, K. (2015). Incorporating GeoGebra into Geometry Learning-A lesson from India. *Eurasia Journal of Mathematics, Science and Technology Education*, 11, 77-86. doi: 10.12973/eurasia.2015.1307a.
- Bordova, E. (1997). Key Concepts of Vygotsky's Theory of Learning and Development. *Journal of Early Childhood Teacher Education*, 18(20), 16-22
- Brousseau, G. (2002). *Theory of Didactical Situation in Mathematics*. USA: Kluwer Academic Publishers.
- Brousseau, G. (1996). Foundations and Methods in Maths Didactics. *Research in Maths Didactics*, 7, 50-51.
- Brousseau, G. (1986). Fondements et Methodes de la Didactique des Mathematiques. *Recherches en Didactique des Mathematiques*, 7, 33-115.
- Burger, W. F. (1981). Interim Report of the Study "Assessing Children's Development in Geometry". *National Science Foundation Science Education Project Director's Meeting*, Washington, DC, AS.
- Brown, S. (2008). Exploring Epistemological Obstacles to the Development of Mathematics Induction. *Proceedings of the 11th Conference for Research on Undergraduate Mathematics Education*. San Diego, CA, hlm. 1-19.
- Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele Levels of Development in Geometry. *Journal for Research in Mathematics Education*, 17(1), 31-48.
- Byrne, R. M. J. (2008). The Rational Imagination: How People Create Alternatives to Reality. *Behavioral and Brain Sciences*. 30(5-6), 439-480. doi: 10.1017/S0140525X07002579
- Cesaria, A. & Herman, T. (2019). Learning Obstacle in Geometry. *Journal of Engineering Science and Technology*, 14(3), 1271-1280.
- Chiocca, C-M. (2006). An Instrumental Obstacle Example. *ICOTS-7*, Salvador, Brazil, hlm. 1-4.
- Çiftçi, Ş. K., & Yıldız, P. (2019). The Effect of Self-Confidence on Mathematics Achievement: The Meta-Analysis of Trends in International Mathematics and Science Study (TIMSS). *International Journal of Instruction*, 12(2), 683-694. <https://doi.org/10.29333/iji.2019.12243a>

- Chevallard, Y. (2005). Teaching and Teacher Training, B. David (Penyunting). Impulses, 4, 215-231. Lyon: INRP
- Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. Dalam D. A. Grouws (Penyunting), *Handbook of Research on Mathematics Teaching and Learning* (hlm. 420-464). New York, NY: Macmillan.
- Clements, D. H., & Sarama, J. (2007). *Early Childhood Mathematics Learning*. in Lester JFK, Ed. *Second Handbook Of Research On Mathematics Teaching And Learning*. New York, NY: Information Age Publishing
- Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education*, 14(2), 83-94
- Connolly, S. (2010). *The Impact of van Hiele-based Geometry Instruction on Student Understanding*. (Disertasi). School of Arts and Sciences, St. John Fisher College, AS.
- Creswell, J. W. (2014). *Educational Research: Planning, Conducting and Evaluating Quantitative and Qualitative Research*. Harlow: Pearson.
- Denzin, N. K. (2012). Triangulation 2.0. *Journal of Mixed Methods Research*, 6, 80–88. doi:10.1177/1558689812437186
- Duatepe, A. (2000). *An Investigation on The Relationship Between Van Hiele Geometric Level of Thinking and Demographic Variables for Preservice Elementary School Teachers*. (Tesis). Middle East Technical University, Turkey.
- Eccles, F. M. (1971). *An Introduction to Transformational Geometry*. Massachusetts: AddisonWesley.
- Edens, K. & Potter, E. (2007). The Relationship of Drawing and Mathematical Problem Solving: Draw for Math Task. *Studies in Art Education*, 48(3), 282-298.
- Ernest, P. (1991) *The Philosophy of Mathematics Education*. London: Falmer Press.
- Flores, A., & Yanik, H. B. (2015) Geometric Translations: An Interactive Approach Based in Students' Concept Images. *North American GeoGebra Journal*, 5(1), 1-19

- Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht, Netherlands: D. Reidel.
- Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele Model of thinking in Geometry Among Adolscents. Dalam N. C. Mathematics. *Journal for Research in Mathematics Education*. 1-198. New York: National Council of Teachers of Mathematics.
- Geddes, D., Fuys, D., & Tischler, R. (1981). An investigation of the van Hiele model of thinking in geometry applied to inner city adolescents. *Research Presession to the National Council of Teachers of Mathematics Annual Meeting*, St. Louis, MO, AS.
- Glaser, B. G. & Strauss, A.L. (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research*. Chicago: Aldine Publishing Company
- Green, D. R., Robinson, C. L. (2009) Introducing GeoGebra to Foundation Year students. *MSOR Connections*, 9(2), 27-30
- Goldenberg, E. P. & Cuoco, A. A. (1998). What is Dynamic Geometry? Dalam R. Lehrer & D. Chazan (Penyunting), *Designing Learning Environments for Developing Understanding of Geometry and Space* (hlm. 351-368). Mahwah, NJ: Lawrence Erlbaum Associates.
- Goos, M., & Spencer, T. (2003). Properties of shape mathematics-making waves, spencer. *The 19th Biennial Conference of The Australian Association of Mathematics Teachers*, Educational Sciences: Theory and Practice, Adelaide: AAMT, hlm. 424–434
- Guba, E. (1981). Criteria for Assessing the Trustworthiness of Naturalistic Inquiries. *Educational Technology Research and Development*, 29 (2), 75-91
- Gutierrez, A., Jaime, A. & Fortuny, J. (1991). An Alternative Paradigm to Evaluate the Acquisition of the van Hiele Levels. *Journal for Research in Mathematics Education*, 22, 237-251. doi: 10.2307/749076.
- Halat, E. (2007). Reform-based curriculum & acquisition of the levels. *Eurasia Journal of Mathematics, Science & Technology Education*, 3(1), 41-49.
- Harel, G. (2010). DNR-Based Instruction in Mathematics as a Conceptual Framework. Dalam *Frontiers*, Bharath, S. & English L (Penyunting),

- Theories of Mathematics Education*, Advanced in Mathematics Education (hlm. 343-368). doi: 10.1007/978-3-642-00742-2\_34
- Harel, G. (2008). What is mathematics? A Pedagogical Answer to A Philosophical Question. Dalam B. Gold & R. Simons (Penyunting), *Proof and Other Dilemmas: Mathematics and Philosophy (Mathematical Association of America)* (hlm. 265-290)
- Harel, G. (1998). Two Dual Assertions: The First on Learning and the Second on Teaching (or Vice Versa). *The American Mathematical Monthly*, 105(6), 497-507.
- Harel, G. & Sowder, L. (2005). Advanced Mathematical Thinking at Any Age: Its Nature and Its Development. *Mathematical Thinking and Learning*, 7(1), 27-50.
- Hoffer, A. (1979). *Geometry. Teacher's Edition*. Menlo Park, CA: AddisonWesley
- Hoffer, A. (1981). Geometry is More than Proof. *The Mathematics Teacher*, 74, 11-18.
- Hohenwarter, M., Fuchs, K., (2004). Combination of Dynamic Geometry, Algebra and Calculus in The Software System GeoGebra. *ZDM Mathematics Education*, 20(70), 1-5. Diakses dari: [http://www.geogebra.org/publications/pecs\\_2004.pdf](http://www.geogebra.org/publications/pecs_2004.pdf).
- Horowitz, G. (2009) It's Not Always Just About The Grade: Exploring The Achievement Goal Orientations of Pre-Med Students. *The Journal of Experimental Education* 78(2): 215–245.
- Howse, T. D. & Howse, M. E. (2014). Linking the Van Hiele Theory into Instruction. *Teaching Children Mathematics*, 21(5), 305-313
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional Reasoning in Nursing Practice. *Journal for Research in Mathematics Education*, 32(1), 4-27.
- Hudson, B. (2008). Didactical Design Research for Teaching as A Design Profession. Dalam B. Hudson, & P. Zgaga (Penyuting), *Teacher Education Policy in Europe: A Voice of Higher Education Institutions* (hlm. 345-364). (Monographs on Journal of Research in Teacher Education). University of Umeå, Faculty of Teacher Education.

- Hudson, B. & Zgaga, P. (2008). *Teacher education policy in Europe: A voice of higher education institutions*. Umeå: University of Umeå
- Hull, A. N., & Brovey, A. J. (2004). The Impact of The Use of Dynamic Geometry Software on Student Achievement and Attitudes Towards Mathematics. *Action Research Exchange*, 3(1), 24-37.
- Jankvist, U. T., Misfeldt, M., & Marcussen, A. (2016). The Didactical contract Surrounding CAS when Changing Teachers in the Classroom. *REDIMAT*. 5(3), 263-286.
- Johnson, A. M., Jacovina, M. E., Russell, D. E., & Soto, C. M. (2016). Challenges and solutions when using technologies in the classroom. Dalam S. A. Crossley & D. S. McNamara (Penyunting), *Adaptive Educational Technologies for Literacy Instruction*, (hlm. 13-29). New York: Taylor & Francis.
- John, S. W. (2011). *Psikologi Pendidikan*. Jakarta: Kencana.
- Jones, K. (2001), Learning Geometrical Concepts using Dynamic Geometry Software. Dalam K. Irwin (Penyunting), *Mathematics Education Research: A catalyst for change*, (hlm. 50-58) Auckland: University of Auckland.
- Jones, K. (2005). Implications for The Classroom: Research on The Use of Dynamic Software. Dalam J. Edwards & D. Wright (Penyunting), *Integrating ICT into The Mathematics Classroom*. Derby: Association of Teachers of Mathematics (hlm. 27-29)
- Juandi, D, Kusumah, Y. S., Tamur, M., Perbowo, K. S., Wijaya, T. T. (2021). A Meta-Analysis of GeoGebra Software Decade of Assisted Mathematics Learning: What to Learn and Where to Go?. *Heliyon*, 7(5). doi: 10.1016/j.heliyon.2021.e06953
- Kalyankar, V. K. (2019). *The van Hiele Analysis of Curricular Materials: A Comparative Study*. (Disertasi). Mathematical Sciences, University of Arkansas, AS. Diakses dari <https://scholarworks.uark.edu/etd/3511>.
- Kandaga, T., Rosjanuardi, R., & Juandi, D. (2022). Epistemological Obstacle in Transformation Geometry Based on van Hiele's Level. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(4). doi: <https://doi.org/10.29333/ejmste/11914>.

- Kansanen, P. (2003). Studying--the Realistic Bridge Between Instruction and Learning. An Attempt to a Conceptual Whole of the Teaching-Studying-Learning Process. *Educational Studies*, 29(2-3), 221–232. <https://doi.org/10.1080/03055690303279>
- Karakuş, F., & Peker, M. (2015). The Effects of Dynamic Geometry Software and Physical Manipulatives on Pre-Service Primary Teachers' Van Hiele Levels and Spatial Abilities. *Turkish Journal of Computer and Mathematics Education*, 6(3), 338–365. doi: 10.16949/turcomat.31338
- Karso, F. P. (2016). *Desain Didaktis Konsep Matriks Transformasi Geometri*. (Tesis). Sekolah Pascasarjana, Universitas Pendidikan Indonesia, Bandung.
- Kemdikbud. (2018). *Kurikulum 2013 Revisi Tahun 2018*. Jakarta: Dirjen Dikdasmen Kemdikbud.
- Keputusan Kepala Badan Standar, Kurikulum, dan Asesmen Pendidikan Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi Nomor 008/H/KR/2022 tentang Capaian Pembelajaran Pada Pendidikan Anak Usia Dini, Jenjang Pendidikan Dasar, Dan Jenjang Pendidikan Menengah Pada Kurikulum Merdeka.
- Kirby, J. R., & Boulter, D. R. (1999). Spatial Ability and Transformational Geometry. *European Journal of Psychology of Education*, 14(2), 283–294. <https://doi.org/10.1007/BF03172970>
- Koehler, M. J., & Mishra, P. (2009). What is Technological Pedagogical Content Knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, 9, 60–70.
- Koehler, M.J., & Mishra, P. (2008). Introducing TPCK. AACTE Committee on Innovation and Technology (Penyunting), *The Handbook of Technological Pedagogical Content Knowledge (TPCK) for Educators* (hlm. 3-29). Mahwah, NJ: Lawrence Erlbaum Associates.
- Kulik, J. A., Kulik, C-L. C., Bangert-Drowns, R. L. (1985). *Effectiveness of Computer-Based Education in Elementary Schools*, 1(1), 59-74.
- Kutluca, T. (2013). The Effect of Geometry Instruction with Dynamic Geometry Software; Geogebra on Van Hiele Geometry Understanding Levels of



- Students. *Educational Research and Reviews*, 8(17), 1509-1518. doi: 10.5897/ERR2013.1554
- Lagrange, J-B., & Hoyles, C. (Penyunting). (2009). *Mathematical Education and Digital Technologies: Rethinking The Terrain*. New York: Springer.
- Leitzel, J. R. (1991). *A Call for Change : Recommendations for The Mathematical Preparation of Teachers of Mathematics*. Washington, DC: Mathematical Association of America
- Lima C., Alves, G, and Soares, A, (2008). Geometric Visualization: How to Acquire It Using Dynamic Geometry Systems?. *2010 Fifth International Conference on Systems*, Menuires, France, hlm. 306-311. doi: 10.1109/ICONS.2008.63
- Maarif, S., Wahyudin, Noto, M.S., Hidayat, W., Mulyono, H. (2018). Geometry Exploration Activities Assisted With Dynamic Geometry Software (DGS) in A Teacher Education Classroom. *Infinity Journal*, 7(2), 133-146. doi: 10.22460/infinity.v7i2.p133-146.
- Maddux, C. B. (1991). Educational Computing Telecommunications: The Road to Maturity, *Educational Technology*, 31(4), 7-9.
- Mainali, B. R. (2014). GeoGebra from Students' Perspectives. *GeoGebra International Journal of Romania*, 3(1), 31-38
- Manno, G. (2006). *Embodiment and a-didactical situation in teaching –learning of perpendicular straight lines concepts*. (Disertasi). Departement of didactic mathematics Comenius, University Bratislava, Slovakia.
- Mariotti, M. A. (2000). Introduction to Proof: The Mediation of a Dynamic Software Environment. *Educational Studies in Mathematics*, 44(2), 25-53.
- Marrades, R. & Gutierrez, A. (2000). Proofs Produced by Secondary School Students Learning Geometry in a Dynamic Computer Environment. *Educational Studies in Mathematics*, 44(4), 87-125. doi: 10.1023/A:1012785106627
- Mason, J. (1999). *Learning and Doing Mathematics. (2nd Revised Edition)*. York, UK: QED.

- Mayberry, J. (1981). *An Investigation of the van Hiele Levels of Geometric Thought in Undergraduate Preservice Teachers*. (Disertasi), University of Georgia, Georgia.
- Mendoza, V., Derling & Nieto, Z. & Vergel, O, Mawency. (2019). Technology and mathematics as a cognitive component Technology and mathematics as a cognitive component. *IOP Conference Series Earth and Environmental Science*. 1414. doi: 10.1088/1742-6596/1414/1/012007.
- Miles, M.B., Huberman, A.M. & Saldana, J. (2014) *Qualitative Data Analysis: A Methods Sourcebook*. Sage, London.
- Minarto. (2017). Penggunaan Aplikasi GeoGebra sebagai Media Pembelajaran dalam Upaya Meningkatkan Prestasi Belajar Matematika pada Materi Fungsi Kuadrat, *Jurnal Mitra Pendidikan*, 1(3), 1-10
- Minichiello, V. (1990). *In-Depth Interviewing: Researching People*. Longman: Cheshire.
- Mishra, P. & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record*, 108(6), 1017-1054.
- Moleong, L. J. (2000) *Metodologi Penelitian Kualitatif*. Bandung: PT Remaja. Posdayakarya
- Molnár, P., & Lukac, S. (2015). Dynamic Geometry Systems in Mathematics Education: Attitudes of Teachers. *ICTE Journal*, 4(19), 19-33. doi: 10.1515/ijjicte-2015-0017.
- Muhadjir, N. (1998). *Metodologi Penelitian Kualitatif*. Yogyakarta: Rake
- NCTM (2006). *Curriculum Focal Points, a report on the topics considered central for school mathematics*. Reston, VA: NCTM.
- NCTM (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
- NCTM. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM.
- Noto, M. S., Priatna, N., & Dahlan, J. A. (2018). Analysis of Learning Obstacles on Transformation Geometry. *International Conference on Mathematics and Science Education (ICMScE 2018), Journal of Physics: Conf. Series*

1157 (2019) 042100, Indonesia, hlm. 1-5. doi: 10.1088/1742-6596/1157/4/042100

- Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). The Learning Obstacles of Pre-Service Mathematics Teachers on Transformation Geometry. *Journal on Mathematics Education*, 10(1), 117–126.
- Nuharini, D. & Wahyuni, T. (2008). *Matematika Konsep dan Aplikasinya*. Jakarta: Pusat Perbukuan Departemen Pendidikan Nasional.
- Nurhasanah, H., Turmudi, & Al Jupri. (2021). Karakteristik Ways of Thinking (Wot) Dan Ways of Understanding (WoU) Siswa Berdasarkan Teori Harel. *Journal of Authentic Research on Mathematics Education (JARME)*, 3(1), 105-113.
- Ofsted. (2002). *Ofsted report: ICT in Schools – Effect of government initiatives*. UK, Retrieved from [http://www.edtechhistory.org.uk/documents/publications/p\\_2002\\_Ofsted\\_ICT\\_in\\_schools\\_2002.pdf](http://www.edtechhistory.org.uk/documents/publications/p_2002_Ofsted_ICT_in_schools_2002.pdf).
- Olive, J. (1998). Opportunities to explore and integrate mathematics with the Geometer's Sketchpad. Dalam R. Lehrer & D. Chazan (Penyunting), *Designing Learning Environments for Developing Understanding of Geometry and Space* (hlm. 395-418). Mahwah, NJ: Lawrence Erlbaum Associates
- Pech, P. (2012). How Integration of DGS and CAS Helps to Solve Problems in Geometry. *17th Asian Technology Conference in Mathematics*, Bangkok, Thailand, hlm. 16–20
- Piaget, J. (1976a). Cognitive Development in Children Piaget Development and Learning. *Journal of Research in Science Teaching*, 2, 176-186
- Piaget, J. (1976b) Piaget's Theory. Dalam B. Inhelder, H. H. Chipman, & C. Zwingmann (Penyunting), *Piaget and His School* (hlm. 11-23). Berlin, Heidelberg: Springer. [https://doi.org/10.1007/978-3-642-46323-5\\_2](https://doi.org/10.1007/978-3-642-46323-5_2)
- Piaget, J. (1977). *The Development of Thought. Equilibration of Cognitive Structures*. Oxford Basil Blackwell
- Pyshkalo, A. M. (1968). *Geometry in Grades 1-4: Problems in The Formation of Geometric Conceptions in Pupils in The Primary Grades*, Moscow: Proresticheuive Publishing House

- Radford, M. (2008). Complexity and Truth in Educational Research. *Educational Philosophy and Theory*, 40(1), 144-157
- Radford, L. (1997). On Psychology, Historical Epistemology, and the Teaching of Mathematics: Towards a Socio-Cultural History of Mathematics. *For the Learning of Mathematics*. 17(1), 26-33
- Sadeghi, M. (2019). A Shift from Classroom to Distance Learning: Advantages and Limitations. *International Journal of Research*. 4(1), hlm. 80-88. doi: 10.29252/ijree.4.1.80
- Sadovsky, P. (2005). The Didactic Interaction with the Procedures of Peers in the Transition from Arithmetic to Algebra: A Milieu for the Emergence of New Questions. *Educational Studies in Mathematics*. 59(1), 85-112
- Saldana, J. (2013). *The Coding Manual for Qualitative Researchers*. Thousand Oaks, California: Sage.
- Şandır, H., & Aztekin, S. (2016). Pre-service Math Teachers' Opinions About Dynamic Geometry Softwares and Their Expectations From Them. *Mathematics Education*, 11(3), 421-431.
- Sahid. (2012). Using ICT to Support RME Mathematics Based Teaching and Learning at Junior Secodary School. Modul diklat SEA Realistic Mathematics Education for Junior Secondary School Mathematics Teachers. *SEAMEO QITEP in Mathematics*. 8(4), 45-58
- Selwyn, N. (1999). Why The Computer is not Dominating Schools: A Failure of Policy or a Failure of Practice?. *Cambridge Journal of Education*, 29(1), 77-91, doi: 10.1080/0305764990290106
- Septian, A, Darhim, & Prabawanto, S. (2020). Mathematical Representation Ability Through Geogebra-Assisted Project-Based Learning Models. *2<sup>nd</sup> ISAMME 2020, Journal of Physics: Conference Series*, 1657 (2020) 012019, doi:10.1088/1742-6596/1657/1/012019
- Setyawan, F., Kristanto, Y., & Ishartono, N. (2018). Preparing In-Service Teacher Using Dynamic Geometry Software. *International Journal of Engineering and Technology*, 7(4), hlm. 367-370. doi: 10.14419/ijet.v7i4.30.22317.

- Shahmohammadi, S. B. (2019). Opportunities and Challenges in Using Dynamic Software in Mathematics Education. *International Journal for Infonomics*, 12(1), 1834-1840.
- Simon, M.A. & Tzur, R. (2004) Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical Thinking and Learning*, 6(2), hlm. 91-104, doi: 10.1207/s15327833mtl0602\_2
- Subchan, Winarni, Mufid, M. S., Fahim, K., & Syaifudin, W. H. (2018). *Buku Sekolah Elektronik Pelajaran Matematika (Edisi Revisi 2018)*. Jakarta: Kemdikbud.
- Sugiyono. (2007). *Metode Penelitian Kuantitatif Kualitatif dan R&D*. Bandung: Alfabeta.
- Sunariah, L. & Mulyana, E. (2019). The Didactical and Epistemological Obstacles on The Topic of Geometry Transformation. *International Conference on Mathematics and Science Education (ICMScE 2019), IOP Journal of Physics: Conference Series. 1521 (2020) 032089*, hlm. 1-7. doi:10.1088/1742-6596/1521/3/032089
- Suryadi, D. (2019). *Landasan Filosofis Penelitian Desain Didaktis (DDR)*. Pusat Pengembangan DDR Indonesia.
- Suryadi, D. (2010). *Metapedadidaktik dan Didactical Design Research (DDR): Sintesis Hasil Pemikiran Berdasarkan Lesson Study*. In Guide Book: Seminar in Math and Science Education (hlm. 54-75).
- Suryadi, D. (2005). *Penggunaan Pendekatan Pembelajaran Tidak Langsung serta Pendekatan Gabungan Langsung dan Tidak Langsung dalam Rangka Meningkatkan Kemampuan Matematik Tingkat Tinggi Siswa SLTP*. (Disertasi). Sekolah Pascasarjana, Universitas Pendidikan Indonesia, Bandung.
- Syahbana, A. 2016. *Belajar Menguasai Geogebra (Program Aplikasi Pembelajaran. Matematika)*. Palembang: NoerFikri Offset
- Taylor, S. J., Bogdan, R., & DeVault, M. (2015). *Introduction to Qualitative Research Methods: A Guidebook and Resource, 4th Edition*. New York; Wiley.

- Tchoshanov, M. (2013). *Engineering of Learning: Conceptualizing e-Didactics*. Moscow: UNESCO Institute for Information Technologies in Education.
- Toom, A. (2006). *Tacit Pedagogical Knowing at the Core of Teacher's Professionalism*. Helsinki: University of Helsinki
- Trouche, L. (2004). Managing Complexity of Human/Machine Interactions in Computerized Learning Environments: Guiding Students' Command Process Through Instrumental Orchestrations. *International Journal of Computers for Mathematical Learning*, 9(281), hlm. 281–307.
- Usiskin, Z. (1987). Resolving The Continuing Dilemmas in School Geometry. Dalam M. M. Lindquist and A. P. Shulte (Penyunting), *Learning and Teaching Geometry, K-12*. Reston, VA, National Council of Teachers of Mathematics.
- Usiskin, Z. (1982). *Van Hiele Levels and Achievement in Secondary School Geometry*. Dalam Final Report of the Cognitive Development and Achievement in Secondary School Geometry Project. Chicago: University of Chicago, Department of Education
- Usiskin, Z. & Senk, S. (1982). *Van Hiele Levels and Achievement in Secondary School Geometry*. Kimbark Avenue: The University of Chicago.
- Vale, I., & Barbosa, A. (2018). Mathematical problems: The advantages of visual strategies. *Journal of the European Teacher Education Network*, 13(3), 23–33.
- Van Hiele-Geldof, D. & Van Hiele, P. M. (1984). English Translation of Selected Writings of Dina Van Hiele-Geldof and Pierre M. Van Hiele. (D. Fuys, D. Geddes, & R. Tischler, Penerjemah), Monograph. New York: Brooklyn College.
- Van Hiele, P.M. (1959). *The Child's Thought and Geometry*, English translation of selected Writings of Dina van Hiele-Geldof and Pierre M. van Hiele, 243-252.
- Van Hiele, P. M. & Van Hiele-Geldof, D. (1958). A Method of Initiation Into Geometry at Secondary Schools. Dalam *Report of The Initiations into Geometry*. H. Fredenthal, (Penyunting). *Subcomitee of the International*

- Commission on Mathematical Instruction for The Netherlands, Report No. III.* Groningen: J. B. Wolters.
- Van Hiele-Geldof, D. (1957). *De didaktiek van de meetkunde in de eerste klas van het V.H.M.O.* (Disertasi), University of Utrecht, Jerman.
- Vojkuvkova, I. (2012). The van Hiele Model of Geometric Thinking. *21st Annual Conference of Doctoral Students, WDS'12 Proceedings of Contributed Papers*, Prague, Czech Republic, hlm. 72–75
- Vygotsky, L.S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press
- Walton, D. (1999). Rethinking the Fallacy of Hasty Generalization. *Argumentation*, 13(2), hlm. 161–182. doi:10.1023/a:1026497207240
- Weber, E., & Lockwood, E. (2014). The Duality Between Ways of Thinking and Ways of Understanding: Implications for Learning Trajectories in Mathematics Education. *The Journal of Mathematical Behavior*, 35(3), 44-57
- Wirszup, I. (1974). Some Breakthroughs in the Psychology of Learning and Teaching Geometry. *The Annual Meeting of the National Council of Teachers of Mathematics*, Atlantic City, NJ.
- Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. Dalam J. L. Martin (Penyunting), *Space and Geometry: Papers from a Research Workshop*. Columbus OH: ERIC/SMEAC
- Zorin, B. (2011). *Geometric Transformations in Middle School Mathematics Textbooks*. (Disertasi). College of Education, University of South Florida, AS.