

**DESAIN DIDAKTIS KUBUS DAN BALOK UNTUK MENGEMBANGKAN  
KEMAMPUAN REPRESENTASI MATEMATIS SISWA SMP**

TESIS

Diajukan untuk memenuhi salah satu syarat untuk memperoleh  
gelar Magister Pendidikan Matematika (M.Pd) pada Program Studi  
Pendidikan Matematika



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### **DESAIN DIDAKTIS KUBUS DAN BALOK UNTUK MENGEMBANGKAN KEMAMPUAN REPRESENTASI MATEMATIS SISWA SMP**

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S.Pd. Universitas Jambi, 2020

Sebuah tesis yang diajukan untuk memenuhi sebagian syarat untuk memperoleh  
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**LEMBAR PENGESAHAN**

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## **ABSTRAK**

**Gita Safitri. (2002088). Desain Didaktis Kubus dan Balok untuk Mengembangkan Kemampuan Representasi matematis Siswa SMP.**

Topik kubus dan balok merupakan salah satu topik matematika yang erat kaitannya dengan kehidupan sehari-hari, hal ini dikarenakan permasalahan mengenai materi ini dapat ditemui dalam kehidupan sehari-hari. Berdasarkan penelitian terdahulu dan studi pendahuluan yang telah dilakukan, pembelajaran yang diterapkan oleh guru hanya berpedoman pada buku teks yang diterbitkan oleh pemerintah sebagai bahan ajar yang digunakan oleh guru, hal ini salah satu penyebab ditemukannya siswa mengalami hambatan belajar. Tujuan penelitian ini adalah untuk merancang desain didaktis luas permukaan serta volume kubus dan balok yang dapat mengembangkan kemampuan representasi matematis siswa SMP. Subjek dari penelitian ini adalah siswa kelas VIII pada salah satu Sekolah Menengah Pertama (SMP) di Kota Jambi, Provinsi Jambi. Analisis data dalam penelitian ini meliputi analisis data sebelum pembelajaran (tahap analisis prospektif), analisis data saat pembelajaran (tahap analisis metapedadidaktik), dan analisis setelah pembelajaran (tahap analisis retrospektif). Hasil penelitian diperoleh bahwa pembelajaran dengan bahan ajar berupa buku teks dari pemerintah berpusat pada guru, sehingga menyebabkan tidak memunculkan adanya situasi aksi, formulasi, validasi, dan institusionalisasi. Pelaksanaan pembelajaran seperti itu berpotensi untuk memunculkan hambatan belajar pada siswa, diantaranya penemuan rumus yang dilakukan oleh guru (hambatan didaktis), kesulitan siswa dalam menyelesaikan soal representasi matematis yang tidak langsung diketahui panjang rusuk kubus maupun balok (hambatan epistemologis), dan rendahnya motivasi siswa dalam mengikuti proses pembelajaran (hambatan ontogenik). Desain didaktis konsep luas permukaan serta volume kubus dan balok hipotesis menekankan untuk siswa dapat berperan secara aktif dalam penemuan konsep, sehingga melalui desain ini dapat meminimalisir hambatan belajar serta dapat mengembangkan kemampuan representasi matematis siswa.

**Kata kunci:** Desain Didaktis, Luas Permukaan Kubus dan balok, Volume Kubus dan Balok, Kemampuan Representasi Matematis

## **ABSTRACT**

### **Gita Safitri. (2002088). Didactical Design of Cube and Cuboid to Develop Mathematical Representation Ability of Junior High School Students**

Cube and cuboid are one of mathematical topics that are related to everyday life, this is because problems regarding these materials can be encountered in everyday life. Based on previous research and preliminary studies that have been carried out, the learning applied by teachers is only guided by textbooks published by the government as teaching materials used by teachers, this is one of the causes of the discovery of students experiencing learning obstacles. The purpose of this research is to create a didactic design of surface area and volume of cube and cuboid that can develop mathematical representation skills of junior high school students. The subjects of this study were class VIII students at one of the Junior High Schools (SMP) in Jambi City, Jambi Province. Data analysis in this study includes data analysis before learning (prospective analysis stage), data analysis during learning (metapedadidactic analysis stage), and analysis after learning (retrospective analysis stage). The results of the study showed that learning with teaching materials in the form of textbooks from the government was teacher-centered, so that there was no situation for action, formulation, validation, and institutionalization. The implementation of such learning has the potential to create learning obstacles in students, including the discovery of formulas made by the teacher (didactic obstacles), student's difficulties in solving mathematical representation problems that are not directly known to the length of the cube or cuboid side (epistemological obstacles), and the low motivation of students in following the learning process (ontogenetic obstacles). The hypothetical didactic design of surface area and volume of cube and cuboid emphasizes that students can play an active role in concept discovery, so that through this design they can minimize learning obstacles and can develop student's mathematical representation skills.

**Keywords:** Didactical Design, Surface Area of Cube and Cuboid, Volume of Cube and Cuboid, Mathematical Representation Ability

## DAFTAR ISI

<b>LEMBAR HAK CIPTA.....</b>	<b>i</b>
<b>LEMBAR PENGESAHAN .....</b>	<b>ii</b>
<b>LEMBAR PERNYATAAN TENTANG KEASLIAN TESIS DAN PERNYATAAN BEBAS PLAGIARISME .....</b>	<b>iii</b>
<b>KATA PENGANTAR.....</b>	<b>iv</b>
<b>UCAPAN TERIMA KASIH.....</b>	<b>v</b>
<b>ABSTRAK.....</b>	<b>vii</b>
<b>ABSTRACT .....</b>	<b>viii</b>
<b>DAFTAR ISI .....</b>	<b>ix</b>
<b>DAFTAR TABEL .....</b>	<b>xii</b>
<b>DAFTAR GAMBAR.....</b>	<b>xiii</b>
<b>DAFTAR LAMPIRAN .....</b>	<b>xv</b>
<b>BAB I PENDAHULUAN .....</b>	<b>1</b>
1.1 Latar Belakang Masalah .....	1
1.2 Batasan Masalah.....	9
1.3 Rumusan Masalah .....	9
1.4 Tujuan Penelitian .....	9
1.5 Manfaat Penelitian .....	10
<b>BAB II KAJIAN PUSTAKA .....</b>	<b>11</b>
2.1 Desain Didaktis .....	11
2.2 Teori Tiga Dunia ( <i>Three Worlds of Mathematics</i> ).....	13
2.3 <i>Design Didactical Research</i> (DDR) .....	14
2.4 Teori Situasi Didaktis (Theory of didactical situation) dan Teori Belajar Pendukung .....	17
2.5 Kompleksitas Situasi Didaktis .....	21
2.6 <i>Learning Obstacles</i> (Hambatan Belajar) .....	24
2.7 <i>Learning Trajectory</i> (LT) .....	26
2.8 Kemampuan Representasi Matematis .....	28
2.9 Penelitian yang Relevan .....	31
2.10 Definisi Operasional .....	32

<b>BAB III METODE PENELITIAN .....</b>	<b>34</b>
3.1 Desain Penelitian .....	34
3.2 Subjek Penelitian .....	35
3.3 Teknik Pengumpulan Data.....	36
3.4 Instrumen Penelitian .....	36
3.5 Teknik Analisis Data.....	38
3.6 Prosedur Penelitian.....	38
<b>BAB IV HASIL PENELITIAN DAN PEMBAHASAN .....</b>	<b>41</b>
4.1 Hasil Penelitian .....	41
1. Analisis Situasi Didaktis pada Konsep Luas Permukaan dan Volume Kubus dan Balok .....	41
2. Analisis Hambatan Belajar Siswa dalam Menyelesaikan Soal Representasi Matematis .....	92
3. <i>Hypothetical Learning Trajectory</i> yang dapat Dikembangkan pada Konsep Luas Permukaan serta Volume Kubus dan Balok ....	127
4. Pengembangan Desain Didaktis Konsep Luas Permukaan serta Volume Kubus dan Balok .....	130
4.2 Pembahasan.....	138
1. Situasi Didaktis pada Konsep Luas Permukaan serta Volume Kubus dan Balok.....	138
2. Hambatan Belajar Siswa dalam Menyelesaikan Soal Representasi Matematis .....	144
3. <i>Hypothetical Learning Trajectory</i> yang dapat Dikembangkan pada Konsep Luas Permukaan serta Volume Kubus dan Balok ....	154
4. Desain Didaktis Konsep Luas Permukaan serta Volume Kubus dan Balok.....	158
<b>BAB V PENUTUP .....</b>	<b>188</b>
5.1 Kesimpulan .....	188
5.2 Rekomendasi .....	192
<b>DAFTAR PUSTAKA .....</b>	<b>194</b>
<b>LAMPIRAN .....</b>	<b>206</b>

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