PENGEMBANGAN KEMAMPUAN PEMAHAMAN, KONSTRUKSI DAN EFIKASI-DIRI BUKTI MATEMATIS MAHASISWA CALON GURU MATEMATIKA MELALUI PEMBELAJARAN BERBASIS-DNR DENGAN TEKNIK EKSPLANASI-DIRI

DISERTASI

Diajukan untuk Memenuhi Sebagian dari Persyaratan
Guna Memperoleh Gelar Doktor Pendidikan Matematika

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ABSTRAK

Iyon Maryono (1402710): **Pengembangan Kemampuan Pemahaman, Konstruksi dan Efikasi-Diri Bukti Matematis Mahasiswa Calon Guru Matematika melalui Pembelajaran Berbasis-DNR dengan Teknik Eksplanasi-Diri**

Memiliki Kemampuan Pemahaman Bukti Matematis (KPBM), Kemampuan Konstruksi Bukti Matematis (KKBM) dan Efikasi-Diri Bukti Matematis (EDBM) yang baik merupakan hal yang penting bagi mahasiswa calon guru matematika. Namun, fakta menunjukkan bahwa KPBM, KKBM dan EDBM sebagian besar mahasiswa masih belum memenuhi harapan. Salah satu masalahnya adalah bagi sebagian besar mahasiswa, pembelajaran belum bisa menjembatani perpaduan antara pengetahuan baru dan pengetahuan lama sehingga menghasilkan struktur pengetahuan yang lengkap. Tujuan utama penelitian ini adalah untuk mengkaji perkembangan KPBM, KKBM dan EDBM melalui model pembelajaran berbasis-DNR (*Duality, Necessity, and Repeated-Reasoning*) dengan teknik Eksplanasi-Diri (DNRED). Penelitian ini menggunakan metode kombinasi dengan desain *embedded* konkuren. Pada tahap kuantitatif digunakan *the pretest-post-test non-equivalent group design* dengan sampel penelitian satu kelompok eksperimen sebanyak 35 mahasiswa dan satu kelompok kontrol sebanyak 34 mahasiswa. Pada tahap kualitatif digunakan pendekatan fenomenografi dengan subjek penelitian kelopok model DNRED sebanyak 35 mahasiswa. Data kuantitatif dianalisis menggunakan Uji-t, ANOVA dua jalur, Uji Chi-Kuadrat dan prosedur pengolahan hasil angket. Tahapan analisis data kualitatif terdiri dari pengkodean terbuka, pengkodean selektif dan pengkodean teoretik. Hasil penelitian menunjukkan, terdapat perbedaan pencapaian dan peningkatan KPBM serta EDBM antara kelompok model DNRED dan kelompok model Ekspositori ditinjau berdasarkan PAG. Terdapat perbedaan pencapaian dan peningkatan KKBM antara kelompok model DNRED dan kelompok model Ekspositori ditinjau secara keseluruhan dan berdasarkan PAG. Tidak terdapat pengaruh interaksi antara dua kelompok model pembelajaran dan tiga kategori PAG terhadap pencapaian KPBM; pencapaian dan peningkatan KKBM; serta terhadap EDBM mahasiswa. Terdapat asosiasi yang signifikan antara KPBM dan KKBM serta antara KKBM dan EDBM, namun tidak terdapat asosiasi yang signifikan antara KPBM dan EDBM. Berdasarkan karakteristik kualitas Eksplanasi-Diri yang diperoleh, pada kelas DNRED menunjukkan adanya perkembangan kemampuan pemahaman, konstruksi dan efikasi-diri bukti matematis.

Kata Kunci: Efikasi-Diri Bukti Matematis, Eksplanasi-Diri, Kemampuan Konstruksi Bukti Matematis, Kemampuan Pemahaman Bukti Matematis, model pembelajaran DNRED

ABSTRACT

Iyon Maryono (1402710): **The Development of Mathematical Proof Comprehension, Construction and Self-Efficacy Ability of Mathematics Teachers Prospective Students through DNR-Based Instruction with Self-Explanation Techniques**

Having good Mathematical Proof Comprehension Ability (MPComA), Mathematical Proof Construction Ability (MPConA), and Mathematical Proof Self-Efficacy (MPSE) is very important for prospective mathematics teacher students. However, the facts show that most students' MPComA, MPConA and MPSE are not as expected. One of the problems is that for most students, learning has not been able to bridge the combination of new knowledge and old knowledge so as to produce a complete knowledge structure. The main objective of this research is to examine the development of MPComA, MPConA, and MPSE through the DNR-based instruction with Self-Explanation techniques (DNRSE instruction model). Mixed methods were used in this study with a concurrent embedded design. In the quantitative stage, the pretest-post-test non-equivalent group design was used with a research sample of 35 students in one experimental group and 34 students in a control group. In the qualitative stage, a phenomenographic approach was used. The research subjects at this stage were 35 students in the DNRSE model group. Quantitative data were analyzed using t-test, two-way ANOVA, Chi-Square test and procedures for processing the results of the questionnaire. The stages of qualitative data analysis consist of open coding, selective coding and theoretical coding. The results showed that based on Prior Geometry Knowledge (PGK), there were differences in the achievement and improvement of MPComA and MPSE between students who received the DNRSE model and students who received the Expository. There are differences in the achievement and improvement of the MPConA between students who received the DNRSE model and students who received the Expository reviewed as a whole and based on PGK. The interaction between two groups of learning models and three categories of PGK has no significant effect on MPComA achievements; achievement and improvement of MPComA; MPSE achievements.There is a significant association between MPComA and MPConA as well as between MPConA and MPSE, but there is no significant association between MPComA and MPSE. Based on the characteristics of the quality of self-explanation obtained, the DNRSE class showed the development of mathematical proof comprehension, construction and self-efficacy ability.

Keywords: DNRED instruction model, Mathematical Proof Comprehension, Mathematical Proof Construction, Mathematical Proof Self-Efficacy, Self-Explanation

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