

**ABSTRAKSI PADA PERKULIAHAN TEORI GRUP:
STUDI FENOMENOLOGI HERMENEUTIK PADA MAHASISWA
CALON GURU**

DISERTASI

Diajukan untuk Memenuhi Sebagian Syarat Memperoleh Gelar Doktor
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**ABSTRAKSI PADA PERKULIAHAN TEORI GRUP:
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CALON GURU**

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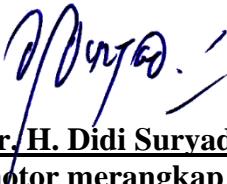
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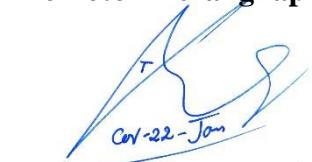


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ABSTRAKSI PADA PERKULIAHAN TEORI GRUP: STUDI FENOMENOLOGI HERMENEUTIK PADA MAHASISWA CALON GURU

Abstrak

Penelitian ini bertujuan untuk menelusuri fenomena abstraksi dan makna-makna objek yang muncul ketika perkuliahan teori grup pada mahasiswa calon guru matematika. Dua hal ini penting dalam aktivitas belajar matematika bagi mahasiswa calon guru yang akan membantu siswa dalam mengkonstruksi pengetahuan matematika. Abstraksi merupakan aktivitas kognitif dalam membangun pengetahuan dengan fokus pada hal-hal yang dianggap penting sebagai dasar pengetahuan baru. Makna objek matematika merupakan representasi dari pemahaman objek matematika yang keluar dari mahasiswa calon guru. Berdasarkan tujuan penelitian, realitas penelitian yang diungkap yaitu tentang fenomena pengalaman belajar (abstraksi) dan makna-makna objek teori grup, maka penggunaan penelitian kualitatif dengan studi Fenomenologi Hermeneutik digunakan sebagai metodologi dalam penelitian ini. Kerangka teori yang digunakan dalam penelitian ini menggunakan teori Triadik Harel yaitu *Mental Act-Ways of Thinking-Ways of Understanding*. Selain teori triadik Harel sebagai kerangka teori besarnya, diperlukan teori Abstraksi Struktural dan Abstraksi Reflektif baik teori APOS, RBC+C, dan teori Reifikasi untuk memahami fenomena abstraksi. Makna objek diperdalam dengan menggunakan teori Konsep dan Objek dari Frege dan Sfard. Partisipan dalam penelitian ini 7 mahasiswa calon guru yang sedang belajar teori grup dikondisikan dengan bentuk aktivitas *Focus Group Discussion* (FGD). Data yang diperoleh berupa aktivitas FGD dari 10 video, 20 audio wawancara pendalaman, dan 7 file pdf catatan harian partisipan. Semua data digital tersebut dikelola, diolah, dan dikoding dengan bantuan aplikasi QDAS Nvivo 12. Temuan dan diskusi hasil dalam penelitian ini tentang aksi mental dan makna objek teori grup pada mahasiswa calon guru. Aksi mental yang muncul pada perkuliahan teori grup diantaranya: aksi kontekstualisasi, aksi menghubungkan pengetahuan baru dengan pengetahuan lama, aksi fokus dan abai pada suatu objek, aksi merampingkan argumentasi, dan aksi manipulasi objek. Makna-makna objek teori grup unik yang ditemukan diantaranya: makna subgrup identik dengan subset, makna invers identik dengan satu per, makna identitas sama dengan menghilangkan elemen, dan makna grup siklik identik dengan elemen berpangkat. Dalam diskusi penelitian, aksi mental yang muncul bisa sebagai tahapan abstraksi dari teori abstraksi yang di teori APOS dan teori RBC+C yaitu aksi kontekstualisasi, fokus dan abai, sintesis pengetahuan, dan merampingkan argumentasi. Aksi kontekstualisasi merupakan aksi mental yang menonjol dalam tahapan abstraksi dan penting disajikan karena berkaitan dengan calon guru yang akan menyampaikan kembali materi tersebut pada konteks yang berbeda. Makna objek yang muncul jika dilihat dari abstraksi memiliki sifat yang operasional dan dinamis. Mahasiswa calon guru harus mampu melakukan abstraksi ketika menyerap pengetahuan dan melakukan kontekstualisasi kembali ketika akan menyampaikan pengetahuannya kembali.

Kata Kunci: *Abstraksi, Makna Objek, Perkuliahan Teori Grup, Mahasiswa Calon Guru, Fenomenologi Hermeneutik.*

ABSTRACTION IN GROUP THEORY LEARNING: STUDY OF HERMENEUTIC PHENOMENOLOGY ON PRE-SERVICE MATHEMATICS TEACHERS

Abstract

This study aims to explore the phenomenon of abstraction and object meanings that appear during group theory learning for pre-service mathematics teachers. These two things are important in learning mathematics activities for pre-service mathematics teachers who will transfer their knowledge to students. Abstraction is a cognitive activity in building knowledge with a focus on things that are considered important as the basis of new knowledge. The meaning of mathematical objects is a representation of the understanding of mathematical objects that come out of pre-service mathematics teachers. Based on the research objectives, the reality of the research that is revealed is about the phenomenon of learning experiences (abstraction) and the meanings of group theory objects, so the use of qualitative research with Hermeneutic Phenomenology studies is used as the methodology in this study. The theoretical framework used in this study uses Harel's Triadic theory, namely Mental Act-Ways of Thinking-Ways of Understanding. In addition to Harel's triadic theory as his grand theoretical framework, Structural Abstraction and Reflective Abstraction theory, both APOS theory, RBC+C, and Reification theory are needed to understand abstraction phenomena. Concepts and Objects theory from Frege and from Sfard to understand more deeply about object meaning. The participants in this study were 7 pre-service mathematics teachers who were studying group theory in the form of Focus Group Discussion (FGD) activities. The data obtained in the form of FGD activities from 10 videos, 6 audio group discussions, 7 audio in-depth interviews, and 7 pdf files of participants' diaries. All digital data is managed, processed, and coded with the help of the QDAS Nvivo 12 software. The findings and discussion of the results in this study are about mental actions and the meaning of group theory objects on pre-service mathematics teachers. Mental actions that appear in group theory lectures include: contextualizing actions, connecting new knowledge with old knowledge, focusing and ignoring an object, streamlining arguments, and object manipulation. The meanings of unique group theory objects found include: the meaning of the subgroup as subset, the meaning of the inverse as one per, the meaning of identity as removing the element, and the meaning of the cyclic group as the rank element. In the research discussion, the mental actions that appear can be as stages of abstraction from the abstraction theory in APOS theory and RBC + C theory, namely contextualization, focus and neglect, knowledge synthesis, and streamlining argumentation. Contextualization action is a mental action that stands out in the abstraction stage and is important to present because it relates to pre-service mathematics teachers who will re-deliver the material in a different context. The meaning of the object that appears when viewed from the abstraction has an operational and dynamic nature. Student teacher candidates must be able to abstract when absorbing knowledge and contextualize again when conveying their knowledge again.

Keywords: *Abstraction, Meaning of Objects, Group Theory Learning, Pre-service Mathematics Teachers, Hermeneutic Phenomenology.*

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