KEMAMPUAN REPRESENTASI SISWA PADA PEMBELAJARAN MATEMATIKA DI SEKOLAH MENENGAH ATAS: SEBUAH KAJIAN DESAIN DIDAKTIS MATERI TRIGONOMETRI

DISERTASI

Diajukan untuk Memenuhi Sebagian dari Syarat
Memperoleh Gelar Doktor Pendidikan Matematika

Oleh

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PROGRAM STUDI PENDIDIKAN MATEMATIKA
FAKULTAS PENDIDIKAN MATEMATIKA DAN ILMU PENGETAHUAN ALAM
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ABSTRAK

Elsa Komala (2022). Kemampuan Representasi Siswa pada Pembelajaran Matematika di Sekolah Menengah Atas: Sebuah Kajian Desain Didaktis Materi Trigonometri

Kemampuan representasi matematis merupakan salah satu tujuan pembelajaran matematika di sekolah yang digunakan sebagai alat bantu untuk menemukan solusi dari permasalah matematika, Tujuan dari penelitian ini adalah untuk mengidentifikasi *learning obstacle* yang terjadi pada siswa dalam pembelajaran matematika di SMA terkait kemampuan representasi serta mengembangkan desain didaktis yang meminimalisir *learning obstacle* tersebut, dan menggambarkan kemampuan representasi setelah implementasi desain didaktis. Penelitian ini menggunakan metode kualitatif dengan desain fenomenologi dan pengembangan bahan ajar dengan *Didactical Design Research* (DDR). Subjek untuk mengidentifikasi *learning obstacle* terdiri dari 34 siswa kelas XI, dan 38 siswa kelas X sebagai subjek pada saat implementasi desain didaktis yang dipilih secara purposive, serta 3 orang guru matematika. Soal tes kemampuan representasi terkait materi trigonometri, wawancara dan studi dokumentasi digunakan pada tahap studi fenomenologi, desain didaktis dan bahan ajar dihasilkan dari proses pengembangan. Data dianalisis secara deskriptif dengan mengkategorisasi berdasarkan jawaban dan pemaknaan yang dihasilkan siswa, kemudian menguraikannya dalam bentuk naratif. Hasil dari penelitian menunjukan bahwa identifikasi *learning obstacle* yang terjadi adalah *ontogenic obstacle* berkaitan dengan keyakinan siswa tentang matematika dan keyakinan tentang dirinya dalam matematika, *didactical obstacle* terkait desain yang kurang mengoptimalkan kemampuan dan kebutuhan siswa dan *epistemological obstacle* berkaitan dengan pengetahuan siswa dalam menyelesaikan masalah kemampuan representasi, *learning obstacle* pada proses visual terkait konsep prasyarat serta memilih informasi yang terkait konsep untuk diilustrasikan, proses membuat pemodelan matematika dengan konteks variasi informasi yang tersedia pada soal, dan pembuatan kata-kata tertulis dengan tidak bisa memberikan penjelasan (alasan) atau kesimpulan pada saat penyelesaian soal. Selanjutnya menghasilkan desain didaktis berupa RPP dan LKS yang dikembangkan dan diimplementasikan, sehingga dapat meminimalisir kesalahan siswa yang terjadi pada proses representasi dengan pemberian intervensi berupa pengintegrasian *peer instruction* dan pemberian *scaffolding*. Hasil dari implementasi desain didaktis menggambarkan bahwa desain didaktis yang dikembangkan dapat meminimalisir *learning obstacle* yang terjadi pada siswa terkait kemampuan representasi matematis, kemampuan representasi matematis siswa yang memiliki dampak besar serta proses yang berkembang paling dominan adalah proses membuat pemodelan matematika, juga proses visual dan pembuatan kata-kata tertulis sudah dilakukan dengan baik oleh siswa.

Kata Kunci: Kemampuan Representasi Matematis, Pembelajaran Matematika, dan Desain Didaktis.

ABSTRACT

Elsa Komala (2022). Students' Representational Ability in Mathematics Learning in Senior High Schools: A Didactic Design Study of Trigonometry

Mathematical representation ability is one of the objectives of learning mathematics in schools which is used as a tool to find solutions to mathematical problems. The purpose of this study is to identify learning obstacles that occur in students in learning mathematics in high school related to representation skills and to develop a didactic design that minimizes the learning obstacle, and describe the ability of representation after the implementation of the didactic design. This study uses qualitative methods with phenomenological design and development of teaching materials with Didactical Design Research (DDR). Subjects to identify learning obstacles consisted of 34 students of class XI, and 38 students of class X as subjects during the implementation of a purposively selected didactic design, as well as 3 mathematics teachers. Representational ability test questions related to trigonometry, interviews, and documentation studies are used at the phenomenological study stage, didactic designs and teaching materials are produced from the development process The data were analyzed descriptively by categorizing based on the answers and meanings generated by the students, then describing them in narrative form. The results of the study indicate that the identification of learning obstacles that occur is ontogenic obstacles related to student's beliefs about mathematics and beliefs about themselves in mathematics, didactical obstacles related to designs that do not optimize students' abilities and needs and epistemological obstacle relates to students' knowledge in solving representational ability problems, learning obstacles in visual processes related to prerequisite concepts and choosing information related to concepts to illustrate, the process of making mathematical modeling with the context of the variation of information available on the problem, and making written words unable to provide an explanation (reason) or conclusion at the time of solving the problem. Furthermore, it produces a didactic design in the form of lesson plans and worksheets that are developed and implemented, to minimize student errors that occur in the representation process by providing interventions in the form of integrating peer instruction and providing scaffolding. he results of the implementation of the didactic design illustrate that the didactic design developed can minimize the learning obstacles that occur in students related to mathematical representation abilities, students' mathematical representation abilities which have a big impact and the most dominant developing process is the process of making mathematical modeling, as well as visual processes and word making. -the written word has been done well by the students.

Keyword: Mathematical Representation Ability, Mathematics Learning, Didactic Design.

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