CHAPTER V
CONCLUSIONS

This study was mainly motivated by two aims: investigating the abstraction process of pre-service mathematics teachers and investigating the role of non-conventional mathematics concepts in the abstraction process of pre-service mathematics teachers. These purposes gave rise to three particular research questions:

1. How do pre-service mathematics teachers' abstraction processes take place when they learn non-conventional mathematics concept?
2. What kind of mathematical abstraction levels that could be raised by pre-service mathematics teachers in learning non-conventional mathematics concept?
3. To what extent the abstraction process of pre-service teachers in learning non-conventional mathematics concepts could indicate their performance in learning conventional mathematics concepts?

In what follows, by focusing on each of these questions, this chapter comprises of three sections referring to the answer of the research questions, and then followed by implication, limitations, and further research.

A. Mathematical Abstraction of Pre-Service Mathematics Teachers in Learning Non-Conventional Mathematics Concept

Related to the first question, researcher not only concerns to "what" concepts constructed by pre-service mathematics teachers but also focus on "how" mathematics concepts are constructed by pre-service mathematics teachers. The elements of knowledge in this study are delivered from the less complex to more complex compound of knowledge elements from E_{A1}, E_{A2}, E_B, E_{C1}, E_{C2}, E_{D1}, and E_{D2}. Based on the data analysis and theory of abstraction levels as the degree of complexity of the mathematical concept of thought in this study, the abstraction process of the complex knowledge elements such as E_{C1} and E_{C2} are promoted through *scaffolding* provided by the lecturer, both instructional and pedagogical. The more complex compound of the knowledge elements, the more *scaffolding* needed to help participants construct the knowledge elements in order to

accomplish the epistemic actions. Without instructional and pedagogical *scaffolding* from the lecturer, the abstraction process pertaining to the more complex knowledge elements could not be accomplished during the learning process in the classroom. Different with the abstraction process of more complex elements, abstraction process of less complex knowledge elements such as E_{A1}, E_{A2}, and E_B mostly taken place in group context. Overall, the abstraction processes in constructing concept of Parallel Coordinates occur in group as well as in classroom context, or viceversa.

Based on the selection process of the focus groups for data analysis in this study, three out of eleven groups were selected to be the focus groups. Two groups are heterogeneous group, and another one is a homogeneous group. Researcher argues that characteristic of the group also plays important role as part of the AiC design. The argumentation behind this response is that the characteristics of the group whether it is homogeneous or heterogeneous will trigger the social interaction between the members of the group. It can also open opportunity for sharing knowledge from the expert participants to the novice participants during the *building-with* action.

Related to the issue of *reducing abstraction* during the abstraction process in constructing Parallel Coordinates concepts, there is a substantial finding resulted from the study that is *reducing abstraction* not always leads to PaCC (*Partially Correct Construct*) condition. In this study, by *reducing abstraction* of the Parallel Coordinates concepts into Cartesian coordinate, participants were successfully solved the problem in Parallel Coordinates. It means that *reducing abstraction* can be used to help participants doing abstraction process through *scaffolding* without leading them to the case of *partially correct construct* (PaCC).

B. Mathematical Abstraction Levels of Pre-Service Mathematics Teacher in Learning Non-Conventional Mathematics Concept

Referred to the data analysis result, the abstraction level of pre-service mathematics teachers in this study can be categorized into three levels of abstraction as presented in Chapter II: level 1 (*perceptual abstraction level*); level 2 (*internalization level*); and level 4 (*second level of interiorization*); but there are

no participant belongs level 3 (*interiorization level*). Researcher also found that some participants do not exactly fulfill certain indicators for certain level; for example some participants are already passed level 2 but not fully fulfill the indicators at the next level. This condition brings up idea to propose the term of "*level in- transition*".

Researcher argues that this finding is related to the action of *consolidation* during the epistemic action that has not been finished. A learner needs to accomplish the epistemic action until *consolidation* of a knowledge element to be able to raise the next level of abstraction. They who in "*transition level*" are participants who pass the *recognition*, *building-with*, and *construction* action but they not come to *consolidation*. Since *consolidation* is part of abstraction process, when participants are recognizing and building-with the new knowledge element in new context, it means that participants who in transition level are not finish doing the abstraction process yet.

Related to the theory of abstraction level as the degree of complexity of mathematical concept of thought, researcher argues that concept of Parallel Coordinates has higher level of abstraction rather than that in the concepts of Cartesian coordinate. Because Parallel Coordinates concept consists of more complex compound and participants need to accomplish the concept of Cartesian coordinate first, before they can constructed the concept of Parallel Coordinates. Using the AiC framework, by defining knowledge elements, the complexity of the knowledge elements is easier to be noticed.

C. Association between Mathematical Abstraction of Pre-Service Mathematics Teachers and Their Performance in Learning Analytic Geometry

The positive correlation between scores of prior knowledge in Cartesian coordinate topic and scores of mathematical abstraction in Parallel Coordinates can be used to ascertain that concept of Cartesian Coordinate could have strong influence for abstraction process in learning concept of Parallel Coordinates. As basic knowledge for constructing the Parallel Coordinates concept, Cartesian coordinate plays significant role in *recognizing* and *building-with* actions.

The positive correlation between scores on mathematical abstraction test and scores on Analytic Geometry test indicated that there is a linear association between both variables. Even though correlation study is not enough to justify that the concept of Parallel Coordinates has significant contribution to scores of Analytic Geometry, the result of this preliminary correlation study can provide invaluable information for future research in investigating the influence of non-conventional mathematics concepts especially the Parallel Coordinates concept for performance of pre-service mathematics teachers in learning Analytic Geometry.

Overall, the study indicates that the abstraction process in learning concept of Parallel Coordinates, as one of non-conventional concepts, has significant contribution to the performance of pre-service mathematics teachers in learning concept of Analytic Geometry. The rational behind this statement is that the abstraction process actually not only takes place in the topic of non-conventional mathematics concept but also in other mathematics concepts.

D. Implications

Implications for mathematics education researcher includes: first, this study used the concept of Parallel Coordinates as one of non-conventional mathematics concepts to be part of AiC design for analyzing the abstraction process that taken place during the learning process. As mentioned by Zaskis (1999), non-conventional mathematics concept can be of any help in constructing richer schemes, developing critical thinking, and providing powerful problem solving tools.

Using the *non-conventional mathematics* concept in AiC design can be beneficial to researcher in defining the knowledge elements and analyzing the abstraction process using RBC model. It can reduce the possible influence of the previous process of concept formation that probably happens when participants learn elementary mathematics concepts. The selection of concepts to be used by researcher helps him/her avoid the possibility of failure in getting data related to the abstraction process. Second, as this study is concern in investigating the abstraction process of pre-service mathematics teachers in learning a *non-conventional* mathematics concept, this study becomes a first step in studying the

role of *non-conventional* mathematics concepts in abstraction process. As a pioneer, this study provides an example of how to: design and use the AiC framework for investigating the abstraction process in learning non-conventional mathematics concepts; use RBC model and reducing abstraction framework for analyzing the data in micro level.

Another implication for pre-service mathematics teachers who involved in this study is that they learn new mathematical concepts which were enrich their mathematics learning experiences. Due to the use of AiC framework, they become rich with experiences in doing mathematical abstraction processes based on context, designed by the researcher. As the concept of Parallel Coordinates is relatively new in mathematics education field, the experiences in constructing the concept enrich them with new learning skills such as *reducing abstraction*.

E. Limitations and Further Research

Up to now, the three research questions set up in this study have been briefly answered completely by the main argument. However, there are some shortcomings and bias of this research study that should be taken into account. The first is related to language translation during the writing part of data analysis. The data in this study were collected in Indonesia, and hence the participants' original verbalization occurred in Bahasa Indonesia, then the verbal data were later translated into English. Despite the fact that the researcher pays considerable attention on the faithfulness of the original content, through translation, some of the properties are inevitably affected. For example, when the English translation of the dialogue is read, it sounds more formal than the original. Yet, the essential meaning between what is written in English and what is actually happened during the conversation remain the same.

The second, realizing that non-conventional mathematics concepts are not accommodated by the curriculum and due to the time constraint, only particular topic in Parallel Coordinates concepts selected as the main knowledge elements that need to be constructed in this study. The knowledge elements involved in this study are limited to the concept of 2D Parallel Coordinates. Those concepts can be related only into small part of Analytic Geometry curriculum. However, because abstraction process actually occurs when someone learns any mathematics

concept for example Cartesian coordinate and straight line, this concept selection is considered appropriate. The concepts that have been selected in this study are considered essential and those concept become basic knowledge for studying any further concepts in Analytic Geometry such as circle, hyperbola and ellipse. In further study, researcher can select more non-conventional mathematics concepts such as hyperbolic geometry for investigating mathematical abstraction process of pre-service mathematics teachers.

The third, considering the time constraints and the efforts required to carry out large scale studies, resulting arguments in this study relied on small number of cases. Researcher believes, it is a good decision. However, although this limitation may raise some questions regarding the substantiations of researcher's arguments, the researcher is cautious not to make hasty generalizations throughout the writing. Furthermore, due to this study aimed to investigate and analyze the abstraction process, not to make generalization, so that detailed analysis such as the ones described in this study is suitable for obtaining reasonably clear understanding of the phenomenon under investigation. It is also appropriate for investigating a relatively new area of research which demands a concurrent inspection of the participants' behaviors and relationships together with an elaboration of the process of mathematical construction. Therefore more theoretical and empirical research is certainly necessary to substantiate or verify the arguments in this study. For the next study, researcher also concern in investigating the relationship between types of groups and types of scaffolding needed during the epistemic actions as the main activities on abstraction process.

Finally, considering that the topic of Parallel Coordinates is relatively new and not accommodated by the curriculum, the researcher decided to serve as the lecturer in this study. At the same time, there are two other lecturers involved in this study. They were taught topic of circle, parabola, hyperbola, and other topics on Analytic Geometry in the same classroom. Although this situation was quite helpful in the course of Analytic geometry, it probably also caused research bias. This bias because she, being the researcher, was rather enthusiastic and highly motivated to facilitate the abstraction process during the instructional process.

However, researcher tried to anticipate it by taken a role as a tutor also for other topics in Analytic Geometry such as circle, ellipse and hyperbola.

Referring to the results of abstraction levels of pre-service mathematics teachers, the relationships among abstraction level of the mathematical concept, the abstraction level of the participants and the concept of reducing abstraction are considered can be investigated in further study. This finding could open opportunity to do further research in exploratory study as well as experimental study.

Based on the result of this study, that there is a correlation between scores on abstraction process test of Parallel Coordinates concept and scores on Analytic Geometry test, the scores of abstraction test on Parallel Coordinates can be used to predict the scores on Analytic Geometry. Although this result is restricted only for a Parallel Coordinates concept, new questions might arise such as "can we improve the content knowledge and pedagogical knowledge of pre-service mathematics teachers by giving them experiences in constructing some non-conventional mathematics concepts?" or "what types of non-conventional mathematics concepts can be used to improve mathematical knowledge of pre-service mathematics teachers?"