# CHAPTER III
# RESEARCH DESIGN AND METHODOLOGY

This chapter provides the methodology used in this study. First, the research questions driven the study are stated. Then it is followed by a detailed description of how this qualitative study was conducted, the research design, a description of the context and its participants. Next, an explanation of data collection processes completed by its sources is described. Then, methods for analysis of the three questions are outlined. Finally, the issue regarding issues of validity, reliability, and trustworthiness for this study concludes this chapter.

## A. Research Approach

Research methodology should be in line with the research questions. Different methodologies come to life when applied to a specific question in a specific context. The studies related to the topic of mathematical abstraction mostly focus on analyzing cognitive processes in constructing new mathematical knowledge. These processes are not easy to capture, so it needs a deep theoretical framework and methodological framework analysis. As the complexity of the research questions is recognized and accepted, it is needed to identify new methods that can capture the complexity in meaningful ways – requiring a "new set of explanations, a new set of tools" (Schoenfeld, 1994).

The first research question focuses on analyzing the processes of mathematical abstraction that occur when pre-service mathematics teachers learn the topic of Parallel Coordinates. This question will be answered by adopting Abstraction in Context (AiC) framework with RBC (*Recognizing*, *Building-with*, and *Construction*) model proposed by Dreyfus, et al (2015) as a theoretical and methodological framework. The details of this method are explained in chapter IV section B on page 65.

The second research question focuses on investigating levels of mathematical abstraction processes. In order to answer this question, the framework from Battista (2007) is used to differentiate levels of abstraction of pre-service mathematics teachers in solving Parallel Coordinates problems, while framework from Hazzan & Zaskis (2005) called as "reducing abstraction" is used to analyze abstraction level of pre-service mathematics teachers based on three interpretations about levels of abstraction as explained in Chapter III, Section 3. Based on this method, a test and a clinical interview serve as a research tool for answering the second question.

The last question that asks whether there is any indication regarding pre-service mathematics teachers performance in learning non-conventional mathematics concept and in conventional mathematics concepts can be more concern with quantitative data, those are scores of test on the topic of Parallel Coordinates and score in analytic geometry so a case study design is addressed for answering the question. The details of the method will be explained in Chapter IV, Section D.

The aims of this study are derived from those research questions, based on the explanation in previous paragraphs; this research belongs to qualitative study using Abstractions in Context (AiC) as theoretical and methodological approach combined with case study design.

In general, this study will have two phases: the first is qualitative study using AiC and RBC model as a methodological tool in phase -1, and then the second is a case study in phase-2. The phase-1 will play its role in order to achieve the first three objectives of this research while phase -2 will do the rest.

**B. Research Procedure**

AiC, as well as any other theoretical frameworks, certainly need methodological tools and method. As a cognitive process that takes place in students' mind, abstraction is not always easy to capture, so a context needs to be designed for studying this process. Dreyfus, et al (2015) stated that the importance of design in research about abstraction is considerable and constrain the kinds of actions participants may carry out, then affords the one that seems desirable to designers and teachers. The design could be an integral part of the research or could be adopted from elsewhere that has been proven successful.

Based on the research design, AiC framework and RBC model, the procedures of this study can be seen in the diagram below:

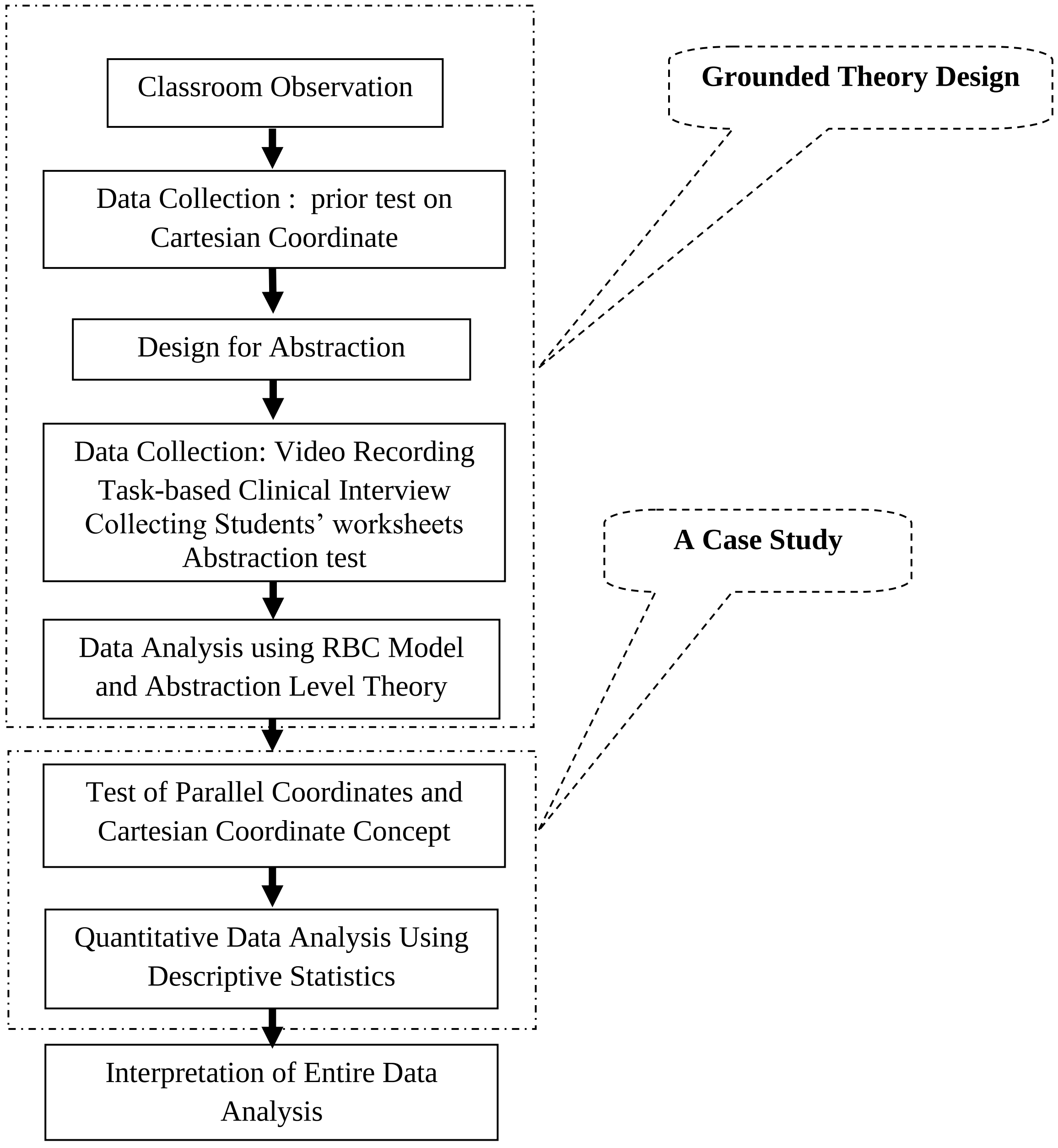


**Figure 3.1**. Research Procedure

In several research studies using AiC to study abstraction, some design principles have been articulated, some of this principles are, triggering a cognitive conflict, asking for hypotheses and providing tools for testing them, and reflective argumentation (Prusak, et al, 2012). Also, the sequence of activities often alternates contexts in which small groups work or argue together with teacher-led reflective discussion (Dreyfus, et al, 2015). Interaction among students in a group is considered as a must to guarantee that the students involved in sharing ideas as well as asking questions to other students that enable them to be involved in the learning process that supports abstraction process.

The next step after setting a design for abstraction process is conducting an a priori analysis. In this step, the assumption about students' previous and prior knowledge is made. The aim of a priori analysis is to identify elements of knowledge as intended by the design such as concepts or strategy of thought. Here also a definition in terms of the mathematical meaning of the element and an operational definition in the context is made. The a priori analysis also works as a working hypothesis that may be confirmed or modified later by the micro-analysis of the data.

## C. Participants and Setting

The participants of this study are 46 pre-service mathematics teachers in the second year of a public university in Bandung who attended Analytic Geometry course. However due to a participant did not meet the attendance regulation, only 45 pre-service mathematics teachers consisting of 40 women and 5 men were involved in this study.

The pre-service mathematics teachers were grouped consisted of 4 until 5 participants in each group. The researcher intentionally used the result of prior knowledge test on topic of Cartesian coordinate to be the basis for grouping process. Based on the result of prior knowledge test, the participants were grouped into three categories, high, medium, and low in term their performance on the prior knowledge test. Using this result, the participants were grouped into 11 groups consisted of 7 homogenous groups and 4 heterogeneous groups. The detail information of each group is described in Table 3.1 below:

Table 3.1. Description of Groups in this Study as A Part of the Context Design

| **Groups** | **Types of Group** | | **Description** |
|---|---|---|---|
| | **Homogenous** | **Heterogenous** | |
| 1 | √ | | High |
| 2 | | √ | High and Medium |
| 3 | | √ | High and Low |
| 4 | | √ | High, Medium, and Low |
| 5 | | √ | Medium and Low |
| 6 | √ | | Low |
| 7 | √ | | Medium |
| 8 | √ | | Medium |
| 9 | √ | | Medium |

| Groups | Types of Group | | Description |
|---|---|---|---|
| | Homogenous | Heterogenous | |
| 10 | √ | | Medium |
| 11 | √ | | Medium |

The entire participants have never been in touch with concept of Parallel Coordinates. They have never learned other types of coordinates such as Polar coordinate. Cartesian coordinate system both in 2D and 3D are the only coordinate systems that they have been familiar with. Their understanding of prior knowledge concepts such as Cartesian coordinate and Lines was varied.

All the participants in this study were given concept of Cartesian coordinate system as part of the curriculum in Analytic Geometry. They also were introduced to Cartesian coordinate through vector approach that consists of topics: (1) Cartesian coordinate; (2) distance between two points; (3) median point; (4) plane vector; (5) directed number; (6) equation of straight lines; (7) bisectors; (8) Parallel and perpendicular lines; (9) distance between point and line; (10) perpendicular bisectors. Those topics were delivered in the first three consecutive weeks.

There were two lecturers who conducted the lecture, a senior lecturer, and a junior one. However, the junior lecturer is the one who was responsible for the teaching while the role of senior lecturer is supervising the learning process of the students. Considering the content curriculum of Analytic Geometry and times constraint, the lecturers and researcher have agreement that topic of Parallel Coordinates would be delivered in additional lecture combined with tutorial class for Analytic Geometry. This tutorial was held once a week. Topic of Parallel coordinates was delivered after the topic of conics in dimension 3. This topic was delivered in three consecutive weeks, completed by a prior knowledge test before the topic was delivered and abstraction test on the topic of Parallel Coordinates after the topic was delivered.

## D. Data Collection Procedures

Five types of data were collected in this study: video recording of the classroom activity; field notes; video recording of semi-structured interviews; documents of students' worksheet; and scores of the test.

Data were collected from the beginning of the Analytic Geometry course. First data were collected through observation of classroom when pre-service mathematics teachers learned concept of Cartesian coordinate. This observation was video recorded; the researcher also made field notes in order to cover sensitive information on classroom activities, particularly when certain data were missing during the examination of video recording.

Three sessions of courses in Parallel Coordinates topic were held after all participants finished learning concept of Cartesian coordinate and did the test for this topic. In these two sessions, all learning processes were video recorded using 8 cameras with two operators. One camera focused on recording the lecturer and the whole class while the other 7 cameras were focused on the selected group who were identified doing some indicators for abstraction such as discussions.

Finishing two sessions of courses, there was a test on the topic of Parallel Coordinates. The test was designed based on the level of mathematical abstraction which was rooted from Piaget (2001), Hershkowitz et al (2001), Battista (2007), Gray & Tall (2007), Nurhasanah (2013), and Hong & Kim (2015).

Reflecting on the result of the test, classroom observation, and students' worksheet, a session of clinical and semi-structured interviews were conducted for 8 participants who have been selected. The interviews were held individually and planned in advance depending on the interviewee's response, allowing unplanned follow up questions, variations on planned questions and clarifying questions (Zaskis & Hazzan, 1999). The clinical and semi-structured interviews were designed based on indicators for mathematical abstraction. It contains questions on the topic of Parallel Coordinates such as the differences and similarities between Cartesian Coordinate and Parallel Coordinates (See Appendix A-3).

The test for mathematical abstraction consists of five problems which are designed based on the levels of mathematical abstraction indicators in topic of Parallel Coordinates. The rubrics of the test were validated through trying out session and experts judgments.

By using AiC framework as methodology in this study, it is necessary for the researcher to design a social and physical setting in order to meet socio-cultural point of view (Herskowitz et al, 2001). The setting in this study was designed using AiC framework. The process of designing learning context based on AiC framework started before the lecture begins, analyzing the curriculum, mathematics topics, the characteristics of the classroom, and the elements of knowledge related to students' activities that would be carried out.

The data were collected within six months period of time through three stages. First, classroom observation, when pre-service mathematics teachers are in the process of learning concept of Cartesian coordinate. The results of the observation were used to complete the design context for abstraction that has been initiated before the lecture started. Second, observation and artifact documented for collecting main data when they learn concept of Parallel Coordinates using AiC design that has been set up. Third, interviews for collecting data took place after the result of the test on Parallel Coordinates has been analyzed.

In addition, for doing a task-based clinical interview, some properties need to be prepared. The most important things are the draft of questions that will be used. According to Zaskis and Hazzan (1999), there are at least six types of questions for collecting data in mathematics education research which are focused on students' thinking: performance questions, unexpected "why" questions, "twist" questions, construction tasks, "give an example" tasks, and reflection questions. Based on the aims of the research, there are at least three types of the questions will be used in this study; they are performance questions, construction tasks, and reflection questions.

According to the result of students' test and classroom observation, there were 8 participants considered fit for interview. They were selected based on their performance in Parallel Coordinates test and their activities during the learning processes.

The second phase of this study needs quantitative data about students' understanding of learning non-conventional concept (Parallel Coordinates) and also conventional concept (Cartesian coordinate). Hence in order to get those data, a set of instruments for both data is needed. The instrument will be set of tests in the concept of Parallel Coordinates and Cartesian coordinates.

**E. Context Design for Mathematical Abstraction**

Referred to AiC methodology as well as a classroom setting, a design that contains activities offering the students opportunities to learn well defined mathematical ideas is also required. What is mathematical knowledge necessary or helpful to create a design suitable for abstraction? The answer is that knowledge should not have been relevant to previous activities carried out by students (Dreyfus et al, 2015).

In this study, while the participants are pre-service mathematics teachers, there are some considerations in selecting mathematical concepts and designing mathematical contexts. In this research, the following considerations have been written constructed for designing context for abstraction based on AiC framework:

1. The concept must be relatively new for pre-service mathematics teachers. They already finish all school mathematics concepts.
2. The concept must not be too advanced for them.
3. The concept must bring benefit for their mathematical horizon knowledge.
4. The concept must be flexible to be embedded in curriculum.

Zaskis (1999) said that pre-service mathematics teachers need to learn non-conventional mathematics concept because it can help them in constructing new mathematical knowledge. More rich and abstract schemes can be constructed during the assimilation process as part of constructing new mathematical knowledge. These concepts are neither part of school mathematics concept, nor part of advanced mathematics concepts.

Suryadi (2016) also stated that pre-service mathematics teachers not only need to learn elementary mathematics concept and undergraduate mathematics concepts such as Calculus, Linear Algebra, Analytic Geometry, and Geometry but also need to learn mathematics concept for graduate level or the result of recent studies from journals or monographs. Through those concepts, pre-service mathematics teachers will have various experiences in thinking processes.

One of the non-conventional mathematics concepts is Parallel Coordinates, a result of recent study in mathematics. Based on the argumentation and considerations mentioned above, the concept of Parallel Coordinates has been selected in this study.

In a priori analysis knowledge elements related to this topic were determined. Parallel Coordinates topic is considered as a new knowledge in mathematics which has already been developed further and had become a new subject that contains many concepts. Although this topic can be taught in one full semester, unfortunately, it is not wise to teach this topic in one semester for pre-service mathematics teachers, because a content of this topic is varied and the development of this concept is dominated by applied mathematics or computer science.

As stated by Inselberg (2009) as the developer of this concept, Parallel Coordinates can be a companion concept in Analytic Geometry. Analytic geometry is a course that must be attended by pre-service mathematics teachers. They learn 2-dimensional and 3-dimensional geometry; Parallel Coordinates can be helpful for them in understanding concept of dimensions and its representation. Based on the curriculum analysis in Analytic Geometry course below are topics related to Parallel Coordinates that have been selected in this study:

1. Construction of Parallel Coordinates
2. Definition of Parallel Coordinates
3. Parallel Coordinates in 2 dimensions
4. Concept of duality in Parallel Coordinates
5. Straight Lines in Parallel Coordinates

All those concepts were written in a module and outlined into lesson activities, then both were tried out to voluntarily participants; they were two freshmen and two sophomore students in mathematics education department. The result of the try out showed that the module is readable and understandable, but it needs minor revisions for grammatical errors and symbols in figures.

The module was completed by lesson activity and students' worksheets. Lesson activities and worksheets were designed for accommodating elements of knowledge in learning activities. The module can be accessed on the Appendix A-2, while the lesson activities and students worksheets are on Appendix A-4 and Appendix A-5 consecutively. The lesson activities and students' worksheets were designed based on AiC framework. The analysis of tasks design as part of the development of lesson activities and students' worksheets are described in Table 3.1 below:

Table 3.2. Design of Learning Activities Based on AiC Framework

| Key Concepts | Prior Knowledge | Specific Instructional Objectives | Design of Activities |
|---|---|---|---|
| 1. Definition of Parallel Coordinates.<br>2. Representation of points and lines in Paralel Coordinates.<br>3. Duality in Parallel Coordinates.<br>4. The position of two or more lines in Parallel Coordinates. | 1. Types of coordinates system in mathematics.<br>2. Definition of Cartesian Coordinates.<br>3. Components of Cartesian coordinates, axes, scales, points and lines<br>4. Differences between 2D and 3D on Cartesian Coordinates. | At the end of the lesson students should be able to:<br>1. represent a point in 2D Parallel Coordinates;<br>2. sketch the graph of $y = mx + b$ in 2D Parallel Coordinates;<br>3. generalize the concept of duality between a point and a line in 2D Parallel Coordinates<br>4. identify two parallel lines in 2D Parallel Coordinates;<br>5. identify two perpendicular lines in 2D Parallel Coordinates;<br>6. analyze the slope of a line in 2D Parallel Coordinates;<br>7. synthesize relationship between the values of $m$ (slope) and the sketch of the graph on 2D Parallel Coordinates;<br>8. evaluate the relationship of the slope of lines and concept of the region;<br>9. generalize relationship between point ↔ line duality and concept of Region in Parallel Coordinates. | Students were divided into groups. Learning activities completed by worksheet and module as part of learning activities designed. Through the worksheet students are expected to do the activities below:<br>1. Constructing Parallel Coordinates using components of Cartesian coordinate;<br>2. Predicting representation of a point $A$ (3,2) in Parallel Coordinates;<br>3. Describing the process of getting the representation of a point in Parallel Coordinates;<br>4. Predicting the representation of a line $y = mx + b$ in Parallel Coordinates ;<br>5. Comparing the representation of a line in Cartesian coordinate and Parallel Coordinates;<br>6. Generalizing the concept of point ↔ line duality. |

All the designs are consulted with two experts in the field of mathematics education and geometry before it was tried out to voluntarily participants. The result from two experts can be accessed in the Appendix A-6 while the result of the try out can be accessed in Appendix A-7.

Based on AiC frameworks, design for abstractions includes creating learning situations such as what tools need to be prepared, or whether students will work in group, individual or in pairs. In this study, group discussion and classroom discussion were combined as part of AiC context. This decision was made based on the result of classroom observation and field notes in the first phase when pre-service teachers learn the concept of Cartesian coordinate. The summary result of classroom observation and field notes in the first phase can be accessed in Appendix B-1.

The classroom was divided into 11 groups of students, where each group comprises at most 5 students. The members of the group were selected by the researcher based on their performance in learning Cartesian coordinate concepts. There were 5 homogeneous groups and 6 heterogeneous groups in terms of their prior knowledge. The characteristic of the groups is varied. The argumentation behind this setting is that it allows the researcher to accommodate all probabilities of social interaction that could take place in classroom learning process.

In order to collect necessary data from the groups, the researcher finds that it is very strategic to observe a group of students that might be expected to perform abstraction process. This group is called the focus group.

Having 11 groups in classroom is very hard to capture the activities of all groups. Therefore, the researcher made use of 8 handy cameras to record learning activities in classroom and asked two skillful assistants for operating those handy cameras. The classroom setting and location of the handy cameras are described in Figure 3.2 below:

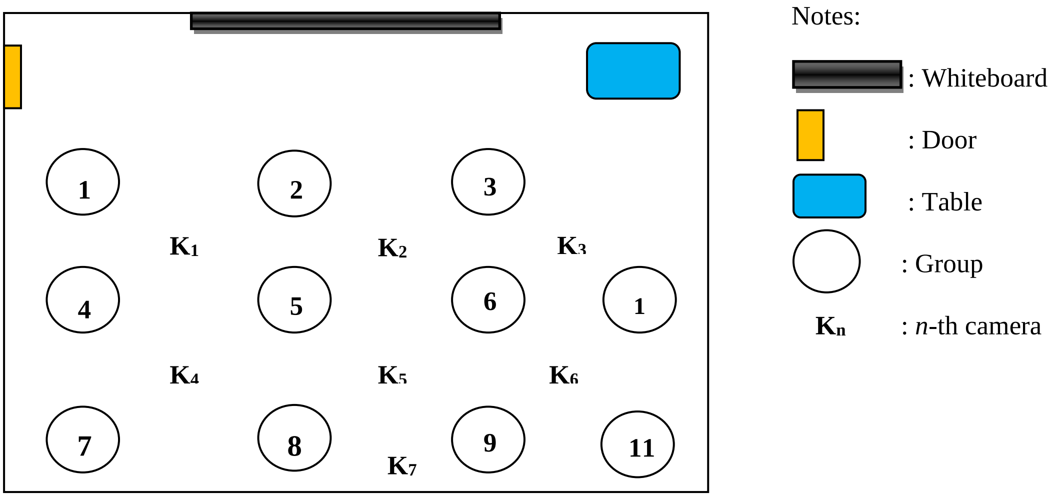


Figure 3.2. Classroom Setting for Collecting Main Data

The characteristic of the focus group is that the group should show mathematical activities and overt expressions. Both characteristics must be identified in the preliminary analysis. Some examples of overt expressions are enthusiasm, uncertainty, surprise, curiosity or bafflement (Dreyfus, et al, 2015). In this study, mathematical activities indicated by activities such as: asking mathematical questions, debating mathematical answers, conducting mathematical talks, answering questions, sketching diagrams or charts and writing answers. The examples of the above overt expressions are added by curiosity face, eye contact, frowning, inclining shoulder and waving hands.

The context for abstraction in this study was completed with problems in the topic of Parallel Coordinates. The problems also served as an instrument for collecting quantitative data. The instrument test consists of 6 essay problems on Parallel Coordinates. The instrument was validated and tried out together with the module and students' worksheets. The reason why involving voluntarily participants for try out of the test instrument in this study is that the participants need to learn the concept of Parallel Coordinates first before they tried to solve the problems.

**F. A Priori Analysis**

This study was designed based on AiC framework with RBC model as a methodology for data analysis, so the terms and codes need to be introduced first before the data analyses were described.

Referred to the design for abstraction stated by Dreyfus, Hershkowitz, and Schwarz (2015), the learning unit should be structured hierarchically in such a way that each stage or task introduces new elements, which evolve from previous stages (globally) and task (locally). Therefore in this study, pre-service mathematics teachers are expected to do construction of four key concepts in four stages: Stage A, Stage B, Stage C, and Stage D.

The pre-service mathematics teachers here were expected to construct the four key concepts that underlie the learning design in this study in four stages: using worksheet 1, 2, 3, and 4, consecutively. Those worksheets were divided into Stage A, Stage B, Stage C, and Stage D.

The relationship of the stages and the key concepts that were constructed in each stage are given below:

Table 3.3. Key Concepts in Each Stage

| No | Stages | Worksheets | Key Concepts |
|---|---|---|---|
| 1 | A | 1 | Construction of Parallel Coordinates in 2D **($E_{A1}$)**<br>A representation of a point $A(x.y)$ in Parallel Coordinates **($E_{A2}$)** |
| 2 | B | 2 | Representation of a line $\ell \equiv y = -2x + 3$ in 2D Parallel Coordinates**($E_B$)** |
| 3 | C | 3 | Generalization of the representation of a line $y = mx + b, m \neq 1$ in 2D Parallel Coordinates **($E_{C1}$)**<br>Representation of a line $y = mx + b, m = 1$ **($E_{C2}$)** |
| 4 | D | 4 | Representation of intersection of two lines **($E_{D1}$)**<br>Representation of two parallel lines **($E_{D2}$)** |

The key concepts are considered here as the main knowledge elements that underlie the learning design and may be expected to be constructed by the pre-service mathematics teachers during the course. A knowledge element in this study is identified with the code $E_x$. Knowledge elements were determined based on key concepts. It is a didactical decision made by the researcher for designing the Parallel Coordinates unit.

Below are the knowledge elements of this study and their explanation:

(1) *Construction of 2D Parallel Coordinates System* **(*$E_{A1}$*)**.

2D Parallel Coordinates is constructed by placing axes in parallel position with respect to axes in 2D Cartesian coordinate system in the plane. The coordinate axes are labeled as $\overline{X}_1$ and $\overline{X}_2$, even the orientation of the axes can be chosen freely but in this study vertical layout is preferred. $\overline{X}_1$ and $\overline{X}_2$ are real line and placed equidistant and perpendicular to *x*-axes in Cartesian coordinate. The distance between $\overline{X}_1$ and $\overline{X}_2$ is labeled as $d$. It can be said that the participants have constructed $E_{A1}$ if they can represent 2D Parallel Coordinates with its components.

(2) *Representation of a point in 2D Parallel Coordinates* **(*$E_{A2}$*)**.

A point $A(x,y)$ in Cartesian coordinate is represented as a line $\overline{A}\ (x_1, x_2)$ in 2D Parallel Coordinates. It can be stated that $E_{A2}$ has been constructed by the participants if they can find correspondence between a point in Cartesian coordinate and a line in Parallel Coordinates. Based on the design, they have to predict and hypothesize the representation of a point $A(x,y)$ using the concept of relation between variable $x$ and $y$, as well as $x_1$ and $x_2$. If the participants assume that a point in Cartesian coordinate corresponds to a line segment or a line, it can be said that they have constructed $E_{A2}$. It can be considered as a reflection of the approach in this study from participants' existing perspective rather than from an expert's perspective.

(3) *Prediction the representation of a line $y = -2x + 3$ in 2D Parallel Coordinates* **(*$E_B$*)**. Making a prediction of line representation on 2D Parallel Coordinates will guide the participants to generalize the representation of any line $y = mx + b$ on 2D Parallel Coordinates. Based on the design, they have to predict and hypothesize the representation of a line $y = -2x + 3$ using knowledge elements of $E_{A1}$ and $E_{A2}$. If the participants found that the representation of line $y$ is a point resulted from at least two points defined on $y$, it can be said that the participants have constructed $E_B$.

(4) *Generalization of representation of line* $\ell \equiv y = mx + b$ with $m \neq 1$ **($E_{C1}$)**. Representation of a line $\ell \equiv y = mx + b$ with $m \neq 1$ in 2D Parallel Coordinates is a Point $\bar{\ell}(\frac{d}{1-m}, \frac{b}{1-m})$ that can be obtained by coordinating and encapsulating $E_{A1}$, $E_{A2}$ and $E_B$. It can be stated that participants have constructed $E_{C1}$ if they can find a point intersection between two arbitrary points on $\ell$ (which are represented as lines) while both are represented in 2D Parallel Coordinates.

(5) *Representation of line* $\ell \equiv y = mx + b$ with $m = 1$ **($E_{C2}$)**. The participants were asked to predict the duality of a line $\ell \equiv y = x + 1$ with $d$ = 3 in 2D Parallel Coordinates and then to generalize the duality of line $\ell \equiv y = mx + b$ with $m = 1$. The participants have constructed $E_{C1}$ if they can find that the representtions is a set of parallel lines which called as *ideal point*.

(6) *The intersection between two lines in 2D Parallel Coordinates* **($E_{D1}$)**. The intersection between two lines in Cartesian coordinate is a point. It can be stated that participants have constructed the $E_{D1}$ if they can find that the intersection of two lines in 2D Parallel Coordinates is a line that passes through the duality of both lines which are points.

(7) *The concept of Duality in 2D Parallel Coordinates,* **($E_{D2}$)**. Representations of two or more lines in 2D Parallel Coordinates are defined by the slope of the lines. It can be stated that participants should have constructed $E_D$ if they can find that two or more perpendicular lines will be represented as different points in different region in 2D Parallel Coordinates and two or more parallel lines will be represented as points in the same region, and the intersection of two lines in Cartesian coordinate is represented as a line in Parallel Coordinates.

All those knowledge elements were constructed in different setting. There was a knowledge element that should be constructed individually, some were constructed in group setting and few were constructed in classroom setting.

The knowledge elements defined in this study should be constructed consecutively from $E_{A1}$ until $E_{D2}$. The hierarchy of the knowledge elements in this design are represented in Figure 3.3 below:

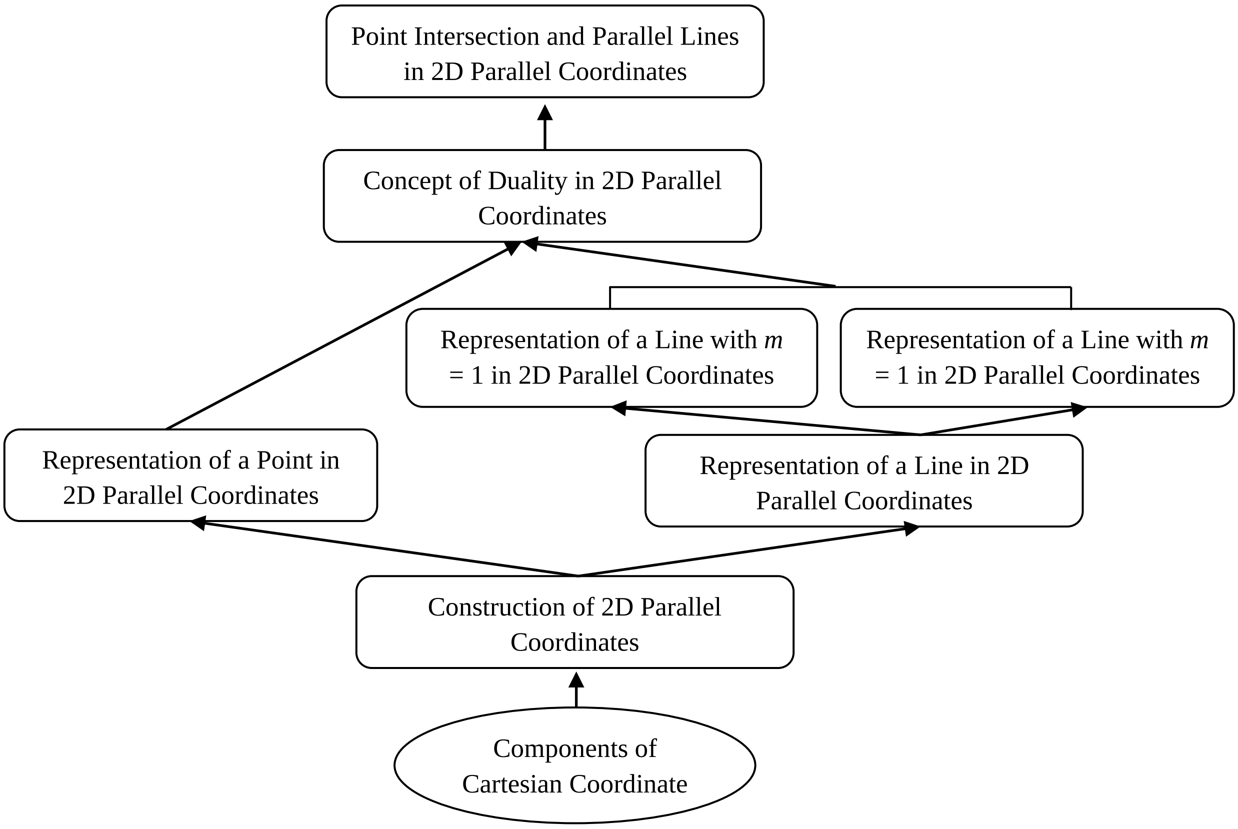


**Figure 3.3.** The Knowledge Elements in this Study

## G. The Process of Data Microanalysis

In line with the aims of this study and the use of AiC framework, data analysis in this study are done in micro-level. Micro-level in social science research referred to individual or small group of individual in particular context, so the microanalysis data in this study means that the data come from dialogue of small amount of individuals (Blalock, 1979), while Staruss & Corbin (1998) defined microanalysis as the detailed line-by-line analysis necessary at the beginning of a study to generate initial categories (with their properties and dimensions) and to suggest relationships among categories; a combination of open and axial coding. In this study, data came from participants' social interaction in a group in the classroom context and the data analysis used a combination of open and axial coding, so the term of microanalysis fits with this study.

In the microanalysis process the researcher did *frame-by-frame coding stage* to select appropriate data, continued b data categorization as the result of frame-by-frame coding stage to do *focused coding*. In this stage, RBC + C model is used to be the tools for analyzing the data using RBC + C table. Finally, the researcher created more general categories; found similarities and dissimilarities as well as patterns among categories; and analyzed the data quantitatively if needed in *analytic coding stage*.

In *Frame-by-frame coding stage* researcher selected appropriate frames from 90 frames resulted in this study, and then coded the selected frames to be analyzed further in focused coding. Actually, the *frame-by-frame coding* in this study was adopted from coding incidents-to-incidents which means the researcher compare incidents to incidents to take hold the ideas then compared it to the code in order to identify properties of emerging concepts (Charmaz, 2006). “The incidents” in this study were recorded in 90 video recording files.

After selecting the frames, the researcher formulated four episodes in order to present the answer to the first question in this study. In every episode, RBC + C method was used to analyze the abstraction process that took place in group context and compared it with the classroom discussion. The researcher provided at most two result analysis from two different groups in every episode.

In order to answer the second research question, the researcher did analysis using data from test result in the topic of Parallel Coordinates and constant comparative technique, which was followed by data microanalysis of interview transcriptions.

## H. Issues of Validity, Reliability, and Generality

Validity, reliability, and generality of qualitative studies do not follow the same rules for design as quantitative studies such as experimental or quasi-experimental studies.  Validity in qualitative research means that the researcher checks for the accuracy of the finding by employing certain procedures; it is based determining whether the finding is accurate from the standpoint of the researcher, the participants, or the readers of an account (Creswell, 2009; Creswell & Miller, 2000). Reliability in qualitative study refers to the consistency of the researcher’s approach across different researchers and different projects(Gibbs, 2007). In addition, the generality in qualitative study often referred to as *transferability* that achieved when the researcher provide sufficient information about self (the researcher as an instrument) and the research context, participants, and the researcher-participant relationship to enable the reader to decide how the finding may transfer (Fraenkel, Wallen, & Hyun, 2012).

One of the criteria for *trustworthiness* adopted in this study is credibility. One key way to address credibility is "the adoption of research methods well established both in the qualitative investigation in general and information science in particular" (Shenton, 2004). In this study, specific procedures that are AiC designed and RBC + C model has been selected from Dreyfus, Heskowitz, & Swartz (2015) used throughout the data collection, analysis, and interpretation in this study. The interview questions also designed based on Zaskis and Hazzan (1999). General qualitative approach and constant comparative analysis were used to ensure credibility as well.

Another strategy used to ensure credibility is triangulation, "cross-checking of data by use of different sources, methods, and at times, different investigators" (Lincoln and Guba, 1986). The researcher used video recorded transcripts, interview transcripts, participants' documents such as their worksheet, test as well as observer field notes to triangulate and verify the findings of this study.

Peer debriefing strategy (Creswell, 2009), "process involves locating a person (a peer debriefer) who reviews and asks questions about the qualitative study so that the account will resonate with people other than the researcher", is another strategy used in this study to ensure credibility. The researcher met with her advisers to consult about emerging design and themes. The researcher also discussed one case thoroughly with peer researchers, who also helped verify the inter-rater reliability of the initial code list.

In order to ensure the consistency of researcher's approach in this study, the researcher documented all processes in detail, and then shared with advisors to help evaluate the processes to confirm the consistency. The researcher also checked the result from three professionals video transcriber to make sure that they do not contain obvious mistake during the transcriptions. Another procedure that used by the researcher is constantly comparing data with the codes and their definitions during the process of coding.

This case study describes results for a small number of participants that are specific to those individuals and environmental contexts. Lincoln and Guba (1986) suggest that it is the researcher's responsibility to provide "sufficient contextual information about the fieldwork sites" to let the reader determine the transferability that also stated by Fraenkel, Wallen, & Hyun (2012).

As recommended by Lincoln and Guba (1986) and Fraenkel, Wallen, & Hyun (2012), the researcher used all strategies suggested by them, by describing in detail the AiC designed used in this study, the participants, the researcher-participant relationship, the data collection techniques, a well as the data analysis process.