CHAPTER I
INTRODUCTION

A. Rationale and Background

The results of international studies such as PISA and TIMSS show that the quality of mathematics education in Indonesia still needs to be enhanced. In 2011, based on the study of TIMSS, the position of Indonesia regarding students' mathematics achievement for grade eight was at 38th position out of 42 participating countries. The mean score of Indonesia was 386, lower than the international mean score which was 500 (Mullis, et al, 2012). In addition, the result of PISA in 2012 also indicated that the position of Indonesia in mathematics was not good; Indonesia was at the 64th position out of 65 participating countries. The mean score was 375, which means 125 points below the whole means.

The quality of mathematics education is influenced by many factors such as curriculum, facilities, learning environment, and teachers. All these factors are considered as integral parts of learning process in a classroom. In classroom teaching process, teacher becomes the key person who is responsible for students' learning process (Turnuklu & Yesildere, 2007; Sabandar & Nurhasanah, 2014). It can be stated that in order to have a good quality of mathematics education we need qualified mathematics teachers.

To obtain qualified mathematics teachers, professional development programs need to be addressed not only for in-service teachers but also for pre-service teachers. In Indonesia, professional development programs for mathematics teachers mostly focused for in-service mathematics teachers. There are at least two institutions that focus on this area they are Educational Institute of Quality Assurance (Lembaga Penjaminan Mutu Pendidikan – LPMP) and Institute for Mathematics Teachers Training (P4TK – Mathematics) (Kusumah & Nurhasanah, 2016).

All mathematics teachers' candidates called as pre-service mathematics teachers are prepared by education institutes called as LPTK (Lembaga Pendidikan Tenaga Kependidikan). LPTK consists of two types of institutions: university (FKIP) and education institute (STKIP and IKIP). Both institutions provide pre-service mathematics teachers content knowledge and pedagogical content knowledge through various subjects, such as Calculus, Geometry, Abstract Algebra, Number Theory, Discrete Mathematics, Differential Equation, School Mathematics, Instructional Design, Models of Teaching, Psychology of Educations etc. By studying all those subjects, pre-service teachers learn mathematical concepts in order to sharpen their mathematical thinking and strengthen their teaching concepts on school mathematics.

Mathematical thinking experiences are very important for pre-service mathematics teachers, as these experiences will contribute to the way they teach mathematics. They can use their experiences in mathematical thinking to predict in which part students will face problem and how they can cope with that problem.

Mathematical thinking consists of various branch such as reasoning, proofing, representation, communication, problem-solving, abstraction, and reflection. All those branch are essential for pre-service mathematics teachers. Some studies related to mathematical thinking in pre-service mathematics teachers are focused on the issues concerning mathematical problem-solving, representation, communication, creative and critical thinking. However, there are still limited studies on abstraction of pre-service mathematics teachers whereas this thinking process belongs to advanced mathematical thinking (Tall, 2002), as well as fundamental process in mathematics and mathematics learning (Ferari, 2003).

Abstraction is important for mathematics, and so is also for teaching mathematics. Fennema and Franke (1992) stated that it is very important for teachers to have mathematical representation knowledge because mathematics is seen as a composition of a large set of highly related abstractions. If teachers do not have knowledge how to translate those abstractions into form that enables students to relate mathematics to what they already know, the students will not learn with understanding. Based on these reasons, it can be concluded that it is crucial for pre-service mathematics teachers to experience abstraction processes in learning a number of new concepts on mathematics.

The idea of abstraction became more popular in mathematics and mathematics education research communities after the commencement of conference for the Psychology of Mathematics Education (PME) in 2002 (Dreyfus and Gray, 2002; Hazzan and Zaskis, 2005). There were three theories of abstraction emerged from that forum, all aimed at providing a means for the description of the processes involved in the emergence of new mathematical mental structures.

Based on studies by experts such as Schwartz, Dreyfus, and Herskowitz (2009); Kidron, 2008; Ozmantar and Monaghan, (2006); Mitchelmore and White (2004); Hazzan (2003), mathematical abstraction could be classified into empirical abstraction and theoretical abstraction referred to types of mathematics concepts in mathematics learning. Empirical mathematical abstraction processes take place when someone learns fundamental mathematics concepts that mostly part of elementary concepts such as numbers and their operation (Mitchelmore and White, 2004). In more advanced mathematics learning, many mathematics concepts have no counterparts in daily life experiences, for example the square root of negative numbers. Those mathematics concepts are easily founded in more advanced mathematics courses such as Real Analysis or Abstract Algebra. The process of abstraction in learning these types of mathematics concepts involves a process of similarity recognition followed by formalization; it is widely known as theoretical abstraction (Mitchelmore and White, 2004; Nurhasanah, 2010).

Studies in the field of new mathematical mental structures which have been done for pre-service mathematics teachers are still limited. Mostly, researchers did the study on elementary level for empirical abstraction topics or in computer science for reflective abstraction. A study did by Hazzan and Zazkis (2005) showed that there were tendencies that pre-service teachers reducing the abstraction level when learning elementary mathematics concepts such as topic of number by connecting the concept with their familiar mental scheme. This phenomenon founded by a researcher when freshman of pre-service mathematics teachers was asked to sketch the graph of $y = x$ in the Cartesian coordinate. Many of the students tended to find the value of x when $y = 0$ and the value of y when $x = 0$ without understanding why they need to do that. They got confused when they found both had equal result (0,0). In this case, they reduced the concept of graph of linear function into a pair of whole numbers in Cartesian coordinate.

Pre-service mathematics teachers need to know abstraction as one of mathematical thinking processes. This process should be a part of their learning experiences and a part of the development of their pedagogical content knowledge in relation to their mathematical knowledge.

As teacher candidates, they have to learn mathematics, school mathematics, and pedagogy for teaching mathematics. Debates about what kind of mathematics that pre-service mathematics teachers need, have led researchers to argue in favor of specific courses and specific strategies for implementing these courses (Zazkis, 1999). Abstract Algebra is one such; through this subject students can learn how to appreciate forms of abstraction and can reach an abstraction level far beyond all its pre-requisites (Dubinsky, et al, 1994; Leron & Dubinsky, 1995). Mathematical modeling is another mathematics concept recommended by Mathematics Association of America (MAA) committee to be thought for pre-service mathematics teachers (Leitzel, 1991). In fact, by considering which mathematics courses are viewed as essential for mathematics teacher certification, different institutions and their academic curriculum designers express their opinion regarding what is seen as an essential background for a mathematics teacher. Courses such as Non-Euclidean Geometry, History of Mathematics and Number Theory are among the frequent choices.

As mentioned by Ball, Thames, & Phelp (2008) there are many of common tasks for teaching that require signifcant mathematical resources such as: presenting mathematical ideas, responding to students' "why" questions, finding an example to make a specific mathematical point, recognizing what is involved in using a particular representation, linking representations to underlying ideas and to other representations, connecting a topic being taught to topics from prior or future years, explaining mathematical goals and purposes to parents, appraising and adapting the mathematical content of textbooks, modifying tasks to be either easier or harder, evaluating the plausibility of students' claims (often quickly), giving or evaluating mathematical explanations, choosing and developing useable definitions, using mathematical notation and language and critiquing its use, asking productive mathematical questions, selecting representations for particular purposes, and inspecting equivalencies. To be able to accomplish those tasks, pre-service mathematics teachers need to develop thinking skills, such as problem-solving, representation, connection, abstraction, and communication.

Mathematical thinking ability is essential for mathematics teachers, because this ability together with content knowledge and pedagogical knowledge support them in developing their pedagogical content knowledge. Moreover, it can be stated that teachers' tendency of thinking will affect students' thinking. Below is a statement revealed by Thompson, Carlson, & Silverman (2007) related to this conjecture:

> "If a teacher's conceptual structures comprise disconnected facts and procedures, their instruction is likely to focus on disconnected facts and procedures. In contrast, if a teacher's conceptual structures comprise a web of mathematical ideas and compatible ways of thinking, it will at least be possible that she attempts to develop these same conceptual structures in her students."

From this statement it can be inferred that teachers tend to use their mathematical thinking in their instructional classroom so students will adopt it the way their teachers did. This means, the information about teachers' mathematical thinking process is very essential for and should be taking into account by researcher in order to analyze students' mathematical thinking which in turn might fully contribute to their mathematical achievements.

Based on the new curriculum issued by the Indonesian government, the focus of mathematics education in Indonesia from now on is moving into problem-solving. As a response to this new policy, pre-service mathematics teachers need to be prepared with all mathematical thinking skills that are relevant to problem-solving activities. In such situation, abstraction becomes one of those which play important role as learning process in mathematical problem solving as studied by Cifarelli (1988). Based on his study, the role of reflective abstraction in problem-solving is helping students in constructing an initial implicit structure for problem-solving and helping students to find different solutions by using different representations.

Abstraction is a process of mental constructions related to the emergence of new concepts. In order to investigate the emergence of new mathematics concepts for future teachers, a consideration must be made in term of what kind of concept that is suitable to this context. Considering that mathematics schools concepts are part of empirical mathematics concepts, all pre-service mathematics teachers learned all those concepts during their elementary and high school time. Those concepts become no longer “new” for them when they enter university.

For those students who have already mastered the basic (empirical) school mathematics concepts as the objects of abstraction before they come to university, learning the same concepts could be an uninteresting activity. Zaskis (1999) even stated that taking the basic empirical school mathematics concepts as the objects of abstraction could be perceived as boring and even insulting. On the other side, preferring to use advanced mathematical concepts in order to explore their abstraction processes also considering as a burden for pre-service mathematics teachers because all those concepts will not be taught to their future students. In addition, because of time constraints, most of curricula for mathematics teachers do not accommodate opportunity to explore mathematical abstraction processes during the classroom teaching.

A study by Ball (1999) showed that having certain ability in solving basic mathematical problems for pre-service mathematics teachers does not always reflect their understanding of the concepts needed for solving those problems. Sometimes they could solve the problems because they memorize the procedures in solving similar problems. However mathematical thinking processes involved in this situation are only memorizing or imitating that considered as lower level mathematical thinking ability.

Ball (1999) provided some evidences from his study that some of pre-service mathematics teachers still have problems in dealing with the basic knowledge of mathematics. Michoux (2013) even stated that many pre-service teachers have no differences with elementary school pupils in terms of their ability in geometry; moreover, they were having similar difficulties and misconception as compared to their pupils.

This condition will cause pre-service teachers encounter problem when they become a teacher and meet smart students who ask questions such as “Why 25 + 5 is equal to 30 not equal to 255?” or “Why we have to put zero point at the intersection of *x*-axis and *y*-axis?”. Without a profound mathematical background, this type of questions will end up with answer “we just take it for granted”.

In order to build a profound mathematical background for pre-service mathematics teachers, a list of mathematics topics is compulsory for them. Even though probably they will have certain preferences but they have to master all compulsory topics. Unfortunately, intensive study about why some mathematics topics are more important than the others for pre-service mathematics teachers are still limited and debatable (Zaskis, 1999).

There are at least three types of mathematics concepts that are important for pre-service mathematics teachers: (1) elementary mathematics; (2) intermediate mathematics; and (3) advanced mathematics (Suryadi, 2016). Elementary mathematics means all mathematics concepts that are learned by students from elementary until high school level. Intermediate mathematics concepts means part of mathematics concept for undergraduate level such as Calculus, Algebra, Linear Algebra, Discrete Mathematics, while advanced mathematics concepts means mathematics concepts that are part of mathematics topics for graduate students or the newest theory from mathematics researches in various journals or monographs. All those types of mathematics concepts can provide complete experiences for pre-service mathematics teachers in mathematical thinking.

Zaskis (1999) has an idea about mathematics concepts that are not part of one of those types stated in the previous paragraph. She proposed a concept of non-conventional mathematics object for pre-service mathematics teachers for achieving deeper understanding of mathematical concepts or constructing richer schemes. Examples of those concepts are fractional number in non-base-10 numeracy, focus-directric coordinate system, Parallel Coordinates, and Boolean operation.

To develop mathematical thinking ability, pre-service mathematics teachers cannot just take the concepts for granted such as from textbooks but they have to experience how the concepts were constructed or developed. Unfortunately, most curricula for pre-service mathematics teachers do not pay enough attention to this issue.

To provide pre-service mathematics teachers with the experience of constructing mathematical concepts, an alternative that can be done is by providing them with learning experiences of mathematics concepts that they have never learned before in order to stimulate the processes of concept construction. Experiences in the processes of constructing new mathematical concepts can sharpen their thinking processes. In addition it also provides them with skills to adapt to new situation.

In order to decide what new mathematics concepts are suitable for abstraction context, some aspects need to be considered. The concepts must not be too rigor for them; it is not part of elementary mathematics concepts but it still can encourage them to use their mathematical thinking; and help them doing abstraction processes. One of the concepts that could be selected is Parallel Coordinates systems as part of non-conventional concepts.

Based on mathematics school curriculum around the world, Cartesian coordinate is one of the compulsory concepts for high school students; almost all high school students are familiar to Cartesian coordinate system. Probably, freshmen in mathematics education department in university also have in mind that Cartesian coordinate system is the only coordinate system that they know before they learning Polar coordinate in Calculus.

Cartesian coordinate system was the greatest invention at the time when for the first time Euclidean geometry could be represented and treated algebraically and numerically. Invented by Rene Descartes (1596 – 1650), the Cartesian coordinate system led to the development of Analytic Geometry which is very useful for surveyors and navigators.

In Analytic Geometry, geometrical ideas are presented using a coordinate system in the Euclidean plane. Cartesian coordinate uses two axes, namely *X – axis* and *Y-axis,* these are two perpendicular number lines of $\mathbb{R}$ where each point on the plane has two coordinates. Here $\mathbb{R}^2$ is the set of ordered pairs of real numbers. Cartesian coordinate system is a basic tool of Analytic Geometry presented in Euclidean plane. Descartes developed it, starting from two-dimensional space in Euclidean plane with $\mathbb{R}^2$ by describing geometry concepts in term of numbers. Then it was extended into Analytic Geometry in three-dimensional Euclidean space identified as $\mathbb{R}^3$. Finally it is generalized into Analytic Geometry in *n* dimensional space ($\mathbb{R}^n$), where the dimension *n* can be any natural number. Unfortunately when $n \geq 4$ for $\mathbb{R}^n$, Euclidean geometry cannot be used anymore to interpret any object in these dimensions in Analytic Geometry.

The concepts and tools developed in Analytic Geometry, particularly on $\mathbb{R}^2$ are very important to be used in generalization and abstraction for multidimensional geometry in $\mathbb{R}^3$, $\mathbb{R}^n$, and Linear Algebra. It is also as a fundamental concept for the notion of dimension in mathematics.

Representing objects in more than 3 dimensions in Cartesian coordinate is complicated. Mostly objects on those dimensions are learned without their representation. This condition leads to difficulties in learning higher dimensions. Study did by Skordoulis, Vitsas, Dafermos and Koleza (2008) found that the limitation on Cartesian coordinate system became epistemological and didactical obstacles for pre-service teachers in Greece in learning the concept of dimensions. The coordinate system becomes an epistemological obstacle when students tend to use the approach with coordinates even though they are working outside of the coordinate system. Cartesian coordinate as didactical obstacles in Greece was identified when many students did not notice the case of interdependence of variables in the case of learning curves equation.

This situation was mainly caused by teaching practice of the Greek mathematics teachers who did not sufficiently analyze the interdependence between variables during teaching. In addition, Greek teachers also did not give emphasis on the parametric equation of the circle and the elips, where the dependence of x and y only one variable.

Based on those situations explained earlier, introducing a new coordinate system could be one of alternatives that can help pre-service teachers to be able to deal with those problems mentioned before. Considering the problems faced by pre-service mathematics teachers in understanding the concept of dimensions, Parallel Coordinates system could be an appropriate concept for helping them to lead their mathematical abstraction process into higher dimensions. This coordinate system can visualize analytic geometry in $\mathbb{R}^n$ (Inselberg & Dimsdale, 1990). Since its first appearance, Parallel Coordinates system has become a well-known visual multidimensional geometry and visualization for exploratory data analysis (Heinrich & Weiskopf, 2012). This non-conventional mathematics concept can help teachers in strengthening and broadening their mathematics knowledge. This concept could also be very useful for analyzing the process of abstraction of pre-service mathematics teachers. This interesting concept, however, has never been introduced to pre-service mathematics teachers.

Theoretical consideration behind this preference is that abstraction is an activity that requires mental construction processes in assimilating new concept. According to Skemp (1971: 46), `*to understand something means to assimilate it into an appropriate schema*'. A question with respect to mathematics teacher education is how can one understand better what has been already understood, that is, assimilated. Skemp (1971) stated that to understand something better means to assimilate it in a richer or more abstract schema. It is suggested that when a new mathematical concepts became in one's mind particular examples of more general mathematical concepts, a richer schema is constructed. This happens, for example, when someone learns integers number; a richer scheme is construdted when he/she comes to an understanding that familiar integers become an example of a commutative additive group.

Information about how pre-service teachers construct new mathematical knowledge could be very useful in order to analyze mathematical thinking process of future teachers. Then, the result could be used to design strategy for developing their pedagogical content knowledge. For example, how pre-service mathematics teachers learn concept of straight lines in Analytic Geometry using vector approach. It can be traced how much the influence of their prior knowledge when learning the concept without vector approach in the concept of Cartesian coordinate.

Study about pre-service mathematics teachers was focused on their achievement or their performance in learning specific concepts in mathematics or focus on what kind of methods or strategies that can be used to enhance their mathematical thinking skills. Unfortunately, studies that have been conducted about their thinking process in learning mathematics are still limited. So that this study will enrich the field of research in mathematics education in term of pre-service mathematics teachers thinking processes. This study will focus on the process of mathematical abstraction that take place when pre-service mathematics teachers learn concept of Parallel Coordinates with the title is "*Mathematical Abstraction of Pre-Service Mathematics Teachers in Learning Non-Conventional Mathematics Concepts*".

B. Research Question

The problem that stands out most, from the background presented above, on one side is the lack of information about how pre-service mathematics teachers abstraction process in learning new mathematics concepts. On the other side, new mathematical concepts learned by pre-service mathematics teachers mostly belong to advance mathematics concepts. In order to fill this gap, this study will document more carefully the process of mathematical abstraction for pre-service mathematics teachers in learning new concepts: non-conventional concept.

Below are some general questions guiding this study:

1. How do pre-service mathematics teachers' abstraction processes take place when they learn non-conventional mathematics concept?
2. What kind of mathematical abstraction levels that could be raised by pre-service mathematics teachers in learning non-conventional mathematics concept?
3. To what extent the abstraction process of pre-service teachers in learning non-conventional mathematics concepts could indicate their performance in learning conventional mathematics concepts?

C. Aims of the Study

The aims of this study, as conducted in this research are to explore the mathematical abstraction processes of pre-service mathematics teachers in learning non-conventional mathematics concepts in analytic geometry using RBC + C model initiated by Schwarz, et al. (2009), to investigate their levels of abstraction based on the theory of level abstraction proposed by Hazzan (1999), Battista (2007), Nurhasanah, Sabandar, & Kusumah (2013), and Hong & Kim (2015). Finally, the last but not least aim of the study is to investigate the relationship between the abstraction processes of pre-service mathematics teachers in learning non-conventional concept and their understanding in learning concept of school mathematics which has similar structure.

D. Terminology

Since this study focused on the topic of mathematical abstraction in non-conventional mathematics concepts, it is really important to explain the definition of terms that are used in this study such as "mathematical abstraction" and "non-conventional mathematics". The term 'abstraction' is used in two different contexts. First 'abstraction' means as a thinking process (also called as 'mathematical abstraction'). Here, abstraction is defined as an activity of vertically reorganizing previous mathematical construct within mathematics and by mathematical means so as to lead a construct that is new to the learner in learning process. This definition refers to Schwarz, et al. (2009). Second, the term of 'abstraction' as ability related to the term of 'reducing abstraction' proposed by Hazzan (2003). This term is based on three different interpretations of *levels of abstraction* discussed in this study: (a) abstraction level as the quality of the relationships between the object of thought and the thinking person; (b) abstraction level as reflection of the process-object duality; and (c) abstraction level as the degree of complexity of the concept of thought.

The term of non-conventional mathematics concepts in this study is adopted from Zazkis (1999). Non-conventional mathematics concept defined as mathematical concepts that are not part of school mathematics and also not part of advanced mathematical concepts. This concept has a similar structure to that of empirical mathematics concepts but not part of mathematics concepts that are learned by students from elementary to high school level. On the opposite, the term of 'advanced mathematics concept' refers to object of mathematics which are not part of empirical mathematics concepts or not part of school mathematics, for example Abstract Algebra, Real Analysis and Geometry from an advance point.

Abstraction in Context (AiC) is a theoretical framework for studying students' processes of constructing abstract mathematical knowledge as it occur in a context that includes specific mathematical curricular and social components as well as a particular learning environment. This framework is completed by a model for describing and analyzing the emergence of mathematical constructs that are new to students. This model called as RBC+C- model which is consist of three observable epistemic actions: *Recognizing*, *Building-with*, *Construction*, and *Consolidation*. This model serves as the main methodological tool for AiC (Dreyfus, et al., 2015).

E. Benefits of the Study

This study will give benefit for at least pre-service mathematics teachers, lecturers in mathematics education department, and researchers in the field of mathematics education. Firstly, through this study pre-service mathematics teachers will benefit in terms of mathematical abstraction knowledge, they will have experiences in doing mathematical abstraction, and they will learn new concept of mathematics that they probably never knew before. They could use all those experiences and knowledge in order to develop their content knowledge and pedagogical content knowledge.

Secondly, this study could help lecturers in mathematics education department to develop their teaching material related to mathematical concepts that must be conveyed to their students. In addition, information about the process of mathematical abstraction of pre-service teachers will be useful for lecturers to determine an appropriate teaching strategy in order to have more effective instructional processes. Knowing more about students' thinking processes can improve their teaching aimed at supporting students' own knowledge construction.

Thirdly, through this study, researchers will have new topics to be explored further. Research on mathematical abstraction is relatively new. This study will use two famous models in topic of mathematical abstraction that have never been used in any study before. It can contribute both for developing mathematical abstraction theory and developing methodology for conducting research in mathematical abstraction topic.

F. Limitation of The Study

This study is not a longitudinal study and, as such, cannot develop curriculum for non-conventional mathematics concepts in various strands in order to lead mathematical abstraction for pre-service mathematics teachers. It would be very interesting indeed to develop such curriculum as Swarchz, et al (2008) did for elementary and high school levels when they constructed the model for abstraction and curriculum standards for mathematical abstraction. However, before a study of such magnitude could be done at higher levels, it would be better if researchers could explore further the tendency of mathematical abstraction of pre-service mathematics teachers and its relationship with their performance in conventional mathematics concepts. Furthermore, this study focused on one single topic at mathematical abstraction in learning non-conventional mathematics concepts, namely "Parallel Coordinates". It does not take into consideration more advanced conception of other mathematical thinking processes occur during learning process.