CHAPTER III
METHODOLOGY

The purpose of this study was to investigate and describe students' algebraic structure sense in order to provide a tentative conjecture on the algebraic structure sense abilities for high school students of Indonesia. The purpose was also to describe strategies that they use and the difficulties that they face in solving algebraic structure sense equations. The current research uses a qualitative case study that is based on the constructivists' perspective that mathematics education should foster the development of students' autonomy and to judge mathematical solutions on the basis of difference and acceptability (Teppo, 1998). The researcher's philosophical view is that students form their own meanings as they learn and meanings are subjective and they differ from individual to individual, (Flavell, 1992). The use of standardized tests alone will not reveal the meanings that individual students might have on any one concept. This notion called for an in-depth inquiry qualitative research methods for the current study.

1.1 Qualitative Case Study

This research is going to use a descriptive qualitative research method with case study design focusing on the study of seven grade 10 students' structure sense in solving algebraic problems. This design was chosen for in-depth exploration of students' algebraic thinking in terms of structure sense. According to Cohen at al. (2007), a qualitative research is a research that aims to understand phenomena about what is experienced by the research subjects, their perceptions, motivations and actions in a natural context. As supported by O'Reilly & Kiyimba (2015, pg. 57), "…the focus of qualitative research is on exploring, examining and describing people…" A case study design according to Suter (2012 pg. 365) is an approach to qualitative research that focuses on the study of a single person or entity using an extensive variety of data. Suter argues that "many descriptive-orientated research questions in education can be answered by intensive study of a single group or similar unit..." (pg. 366). In this case, similar unit are seven students who would show a variety of algebraic structure sense

abilities.

1.2 Place and Time

The current study was carried out at Sekolah Menengah Atas Laboritorium Universitas Pendidikan Indonesia (Lab School UPI) in Bandung, Indonesia. The choice of place was based on the convenience of the research field to the researcher such that transport and other costs are kept at minimum. Lab School was chosen because it uses the Indonesian National Curriculum (Kurikulum 2013) whose aims and objectives of the curriculum suit the issues in the current study about developing and accessing students' mathematical skills. The time scheduled for data collection and analysis is from late November 2018 to late January 2019. Data was collected during the times scheduled by the school to avoid any inconveniences in the normal business of the school and was collected during out of timetable sessions.

1.3 Research Subjects

Subjects from this study comprised of twenty-four $(n = 24)$ grade 10 students from UPI lab School who have studied algebraic concepts. Grade ten students were chosen because those were the ones who have studied algebraic expressions and equations since junior high school and are also going to face university algebra which greatly require aspects of structure sense ability after they complete senior high school. The subjects comprised of 13 girls and 11 boys who were aged 15-17 years. Seven students have been selected for in-depth analysis through audio recorded interviews based on a variety of structure sense ability, strategies and difficulties that they have displayed in the pencil-and-paper algebraic structure sense test.

1.4 Data Sources

In qualitative research, usually the researcher plays a major role in the whole process of data collection. The researcher him/herself was the instrument of the research (Crook, Halsey Bullen, & Todd Johnson, 2004) together with the help of mathematics teachers. Data collection techniques in this research involved giving an algebraic structure sense pencil-and-paper test on algebraic equations to twenty-four grade ten

students, holding interviews and document studies.

1.4.1 Test

An algebraic structure sense paper-and-pencil test was administered to 24 research subjects and was completed in a period of 60 minutes. A test according to Frankel (2012) is an item or a procedure that is used to know or measure a certain phenomenon in question with a way that has already been determined, (Fraenkel et al. 2012). A test is regarded as unobtrusive data source, since analysis will be based on students' working processes. Cohen et al. (2007, pg. 414) describes that a diagnostic test is an in-depth test to discover particular strengths and weaknesses or difficulties that subjects experience, in this case, during solving equations.

The tests for this research stood from 8 questions on algebraic linear and quadratic equations of one variable formed with the belief that the questions are suitable to expose students' algebraic structure sense and strategies in solving equations (Index 8). Test questions were adapted and modified from students' mathematics book grade 10, from Indonesia National Examination past papers for Indonesia (UN 2009) and from other related research instruments used before. The test was given to students who have learned quadratic expressions and equations.

The purpose of giving the test to subjects is to first assess their **algebraic structure sense** in the following manner:

1. Whether or not a student can recognise a simple structure (linear and quadratic) as a previously met structure.
2. To assess whether a student is able to deal with a term as a single entity and make proper substitutions to solve a simplex problem.

Example item question 2

Solve the equation: $7(2p + 5) = (2p + 5)$

The item is set to assess the ability of the students to deal with the term $(2p + 5)$ as a single entity and apply the concept of equivalence relations to solve the problem. Students were expected to subtract $(2p + 5)$ from both sides of the equation in the following manner:

$$7(2p+5)-(2p+5) = \quad (2p+5)-(2p+5)$$
$$6(2p+5)=0$$
$$2p+5=0$$
$$2p+5-5=0-5$$
$$2p=-5$$
$$p=-\frac{5}{2}$$

3. If a student can divide an entities into substructures by manipulating i) a numerical structure and ii) an algebraic structure to make best use of structure through recognizing mutual connections between structures and recognises which manipulations are possible to perform.

   Example question 3i): Manipulating a numerical structure

   Evaluate without using a calculator:

   $$450 \times 13 - 450 \times 3 = 450(13-3)$$
   $$= 450(10)$$
   $$= 4500$$

   Example question 3ii): Manipulating an algebraic structure

   Find $\boldsymbol{m}$ from the following equation:

   $$(m^2-2m)^2=(m-2)^2$$
   $$m^2(m-2)^2=(m-2)^2$$
   $$m^2(m-2)^2-(m-2)^2=(m-2)^2-(m-2)^2$$
   $$(m-2)^2(m^2-1)=0$$
   $$(m-2)(m-2)(m+1)(m-1)=0$$
   $$m=2, m=2 \; or \; m=-1 \; or \; m=1$$

   (Appendix 9)

Secondly, the test is given in order to describe how flexible are students in solving algebraic structure sense equations at grade 10. A student who solved the problem with multiple ways of unique/self-invented ways showed algebraic flexibility. Finally, the test shall act a tool to expose students' difficulties in algebraic learning activities viewed from structure sense ability. Algebraic difficulties were identified in view of the following aspects; errors (computational errors, omissions and errors in transfer), misunderstanding equivalence relations and variables, errors in algebraic

manipulation which involve the application of algebraic operations (associative, distributive and commutative properties).

Before questions were given to the research subjects were validated first. Criterion validity of the test questions was done by consulting experts, who are the researcher's supervisors to make sure questions presented exhibit students' structure sense ability. Research supervisors determined the face validity of the test paper. This was done to ensure that words that are used in the questions are suitable (Appendix 7) and the appearance and format of the test is suitable. Content validity was done by qualified mathematics teachers at the place of research to make sure that algebraic equations presented in the test are suitable for students in grade 10 and are in the current curriculum that is being used at school. Qualified mathematics teachers were requested to give their comments and suggestions on the suitability of the test questions set in the view of algebraic structure sense.

Table 3.1
Item Validation Sheet by Mathemataics Experts

ITEM	DESCRIPTOR	YES	NO	EXPERTS' SUGGESTIONS
1	Question can be used to measure the students' conceptual understanding of algebraic expressions/equations.			
2	Question can be used to measure students' structure sense in identifying simple structures embedded in a complex structure in the problem.			
3	Question can be used to identify students' expert strategies (diversified method) in solving the algebraic problem			
4	The instruction from the question is clear and not ambiguous.			

Each item shall be validated using an item descriptor shown on Table 1. The

descriptor has been adapted and modified from Sari (2014) which was used to investigate grade students' difficulties in algebraic understanding. The researcher corrected and discards questions that have been indicated by mathematics experts as not valid and adjusted questions according to experts' suggestions (Table 3.1).

3.4.2 Interviews

Data collected by pencil-and-paper test was supported by in-depth follow up interviews upon seven of the twenty-four participants who displayed a variety of algebraic structure sense ability and difficulties in the pencil-and-paper test. Interviews were important in understanding students' thinking and working process as well as assessing conceptual understanding. The researcher used an interview guideline (Appendix 14) which was helpful in reflecting on what was to be explored from the subjects' thinking processes. Questions were based on the interviewee's working process as well as the interviewee's response after he/she is posed a question. It is argued that words that are said and actions that are done during the interview are an important source of data (Allsopp, Lovin, & Ingen, 2015), hence the necessity of taking every word said and every action done by the interviewee seriously..

Interview transcripts were recorded by an audio system and were analysed to triangulate data from written test. This was done to explore the students' minds concerning structure sense. Seven interview sessions lasted between 20 to 40 minutes, excluding time where the researcher was explaining some concepts when students have shown some kind of misunderstanding of a certain concept. Only one interview lasted for 18 minutes. At least two subjects were interviewed per day until all the seven subjects went through the interview session. Interviews were conducted during the time the students were not attending any class or during the time that was permitted y the teacher.

3.4.3 Document studies

Teaching and learning records as well as social records are important sources of unobtrusive data. Records give evidence of what has happened before the research process and they help in focusing on the researched problem (Suter, 2012). In this research, the researcher studied the textbooks that are in use at grade 10 as well as the

curriculum document. Studying the textbook helped in getting an insight on students source of errors and methods of solving equations. Studying the curriculum document was aimed at finding the necessary material that are supposed to be given to grade ten and the requirements of the curriculum.

3.5 Research Procedures

The purpose of this research is to provide a tentative hypothesis on Indonesian High School students' algebraic structure sense and identify the difficulties that students face in solving these problems. In order to achieve these goals, the following are the formal processes that were followed in this research:

a. The researcher found literature on algebraic concepts and algebraic structure sense and identified indicators of structure sense.
b. Analysed concepts on algebraic equations and compiled a test that was suitable in identifying students' structure sense, strategies and difficulties as they work on the test questions.
c. Seek formal entry into the research field by presenting a formal introductory letter to the headquarters of the school, which in turn granted permission for the researcher to enter into the desired field: UPI Lab School (Appendices 2 and 3) Research subjects were explained the purpose of the research and were told the day that they were going to write the test. Research subjects and one mathematics teacher of grade ten signed consent forms as a sign that they were willing to participate in this research (Appendices 5 and 6)
d. The researcher did a general survey of the school to get to familiarize with the school system as well study the material used at the school which are include curriculum documents and textbooks.
e. The researcher discussed and formulated structure sense test with mathematics experts including the mathematics teacher based on question validation criteria (Table 3.1) and discussed and adjusted according to experts' suggestions.
f. Research subjects were given an algebraic structure sense test which was analysed according to correct/incorrect answers, strategies used by students to answer the questions, how flexible where the students in answering the questions and what are

the difficulties shown.

g. Seven subjects were chosen for interview as a follow up of the algebraic structure sense and difficulties shown in the test and results were analysed accordingly.

h. The researcher also made an analysis of the textbook and syllabus which are used for the teaching and learning of algebraic equations at the school (Figure 3.1).

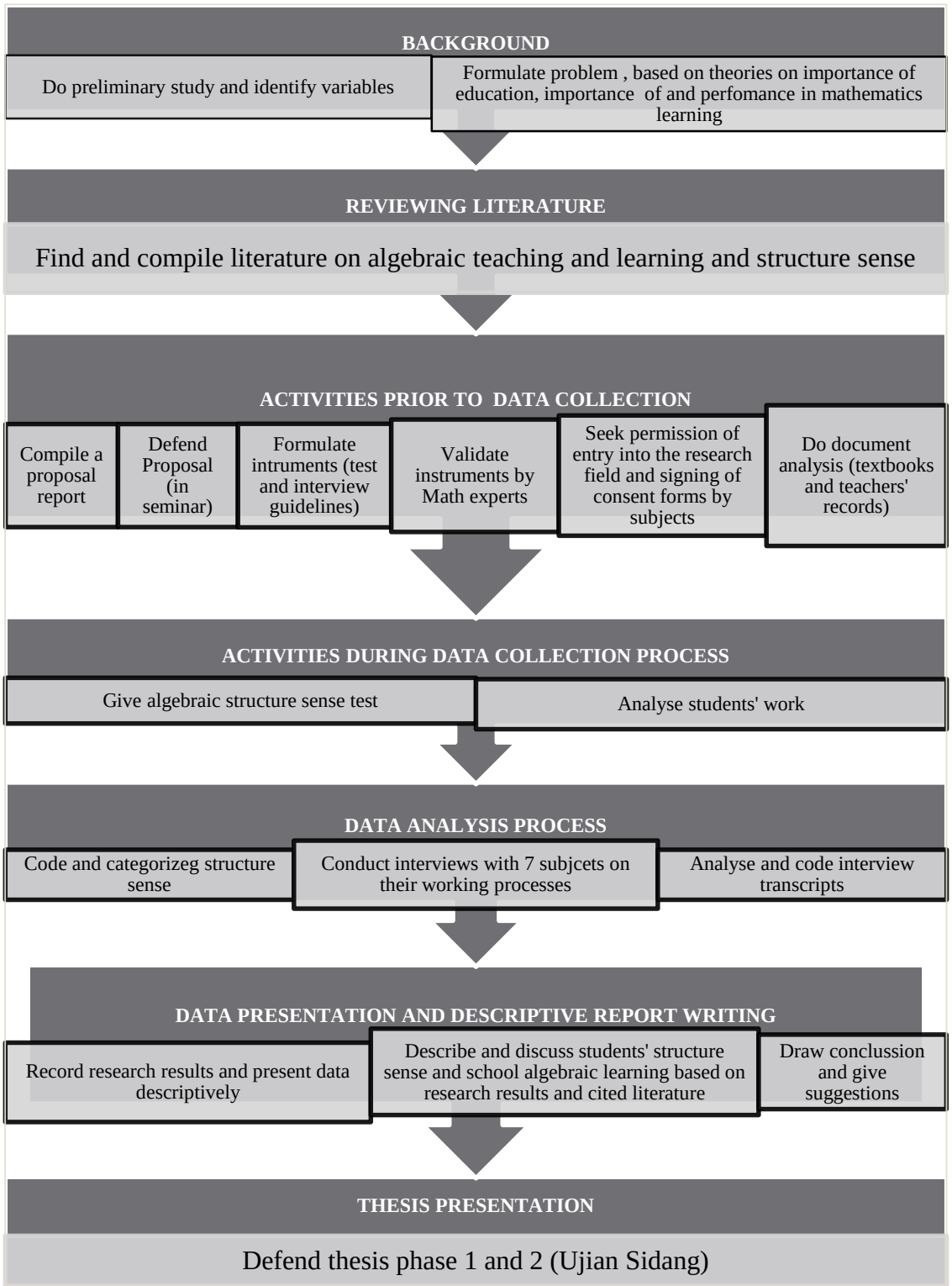

Figure 3.1 Research Procedure

3.6 Data Collection Procedure

A structure sense pencil-and-paper test was administered by the researcher in the month of December 2018 to 24 grade ten students in which students were asked to solve liner and quadratic equations as individuals in a period of 60 minutes. The students were **not** allowed to use any electronic devices in the process of computing data. This instruction was explicit for them. Each subject was presented with a question paper, a pen and an answer sheet and was requested to show their working on the answer sheets provided. Subjects were encouraged to present their working processes in several ways possible and to present it neatly and eligibly. The researcher invigilates the students and was attending to their questions where they were seeking clarity. The researcher took answer sheets for analysis. Students' work was analysed, coded and categorised structure senses, flexibility and difficulties according to the coding system in **Table 3.2.**

In January 2019, seven subjects were chosen for interviewing based on their working processes, a variety of structure sense abilities and difficulties that they have shown in their panic-and-paper test. The interview transcripts were audio recorded. During interviews students were encouraged to name the nature of an equation (wether it is linear or quadratic), to explain how they obtained their solutions and to give reasons for not answering some questions. Interview data was analysed, coded and categorised depending on their similarities. Subjects were also given the chance to ask questions to the researcher after the researcher finished interviewing them.

3.7 Data Analysis Procedure

Data analysis in this study was be done by constant comparison, coding and categorization of similar themes. Data analysis in descriptive case studies according to Cohen et al. (2007) is a process of organising data into patterns, category and single basic file in order to get a theme or a working hypotheses based on what the data suggests. In descriptive qualitative research, analysis is done throughout the research process from data collection to analysis before, during and after leaving the research area. The idea of constant comparison analysis process is coined by Glaser (2013) and the same idea shall be used to analyse students' structure sense. Data was constantly

compared and categorized depending on data similarities with the purpose of identifying students' structure sense abilities, flexible strategies and difficulties in solving algebraic structure sense equations.

Analysis was based on the indicators of structure sense that are mentioned earlier in Chapter 1. The system of coding has been adopted and modified from the works of Hoch & Dreyfus (2006) as well as Jupri & Sispiyati (2017). Four aspects were looked at, for each item: correct/incorrect solution, structure sense abilities **(SS1, SS2, SS3a, SS3b)** against procedural strategy (**PS**), flexible strategy **(FS1 and FS2)** and algebraic difficulties (Computational and omission errors, understanding of variables and equivalence relations, difficulties in algebraic operations and emotional difficulties). Research Subjects were coded; for example **P10** for subject participant number **10**. Correct/Incorrect Solution, Structure Sense Data and Flexibility data from written work were computed using Microsoft Excel Version 2010. The reason to analyse for correct/incorrect answers is to determine student's algebraic abilities before any other aspect would follow.

3.7.1 Analysing Correct/Incorrect Answers

First, analysis was done on identifying the correct and incorrect solutions. The correct Solutions were coded **(1)** whist the incorrect are to be coded **(0)**, on the other hand, question items which are not attempted are coded **X.** These codes for correct and incorrect solutions were researcher invented codes, however the basic idea was adopted from Kokasih (2017)'s UPI dissertation on students' difficulties in understanding congruency and similarity in geometry.

3.7.2 Analysing Structure Sense Abilities

Structure sense activities were defined in four classes. Structure sense abilities were identified regardless of the fact that the final solution is correct or incorrect or the method is formal or informal. First, structure sense was identified in a general form, regardless of the specific indicator for each item. A student who displays structure sense ability of any nature was coded (coded **SS**), that is the one who treat collective terms in solving a complex term rather than expanding/removing brackets otherwise it is procedural **(**coded **PS).** Furthermore, activity, words and action that shows that student

is able to 1) recognise structure in its simplest form was coded **SS1**, 2) if a student is able to deal with a compound term as a single entity was coded **SS2** 3) If one chooses appropriate manipulations to make best use of structure in a numerical form the coded was **SS3a,** 4) If a student is able to choose appropriate manipulations to make best use of structure in an algebraic problem, this was coded **SS3b (Table 3.2)**.

Table 3.2
Coding System for Algebraic Structure Sense

Code	**Ability Example Comments**	**Algebraic Structure**	**Example Question**
SS1	Recognise structure in its simplest form	$\boldsymbol{ax + b = 0}$ (Linear structure) $\boldsymbol{ax^2 - bx + c = 0}$ (quadratic structure)	Find $(2y + 4)^2$ from the equation: $2(2y + 4)^2 - 9 = (4y^2 + 16 + 16y) + 5$
SS2	Deal with compound term as single entity.	$\boldsymbol{a.b = b}$	Solve the equation: $(2m + 5)(m - 3) = m - 3$
SS3a	Choose appropriate numerical manipulations to make best use of structure in simplest form (Recognise the structure and the advantage of factoring)	$\boldsymbol{xa - xb}$ (linear numerical structure) $\mathbf{a^2 - (a + 2)(a - 2)}$ (quadratic numerical structure)	Evaluate without using a calculator: $9745^2 - 9747 \times 9743$
SS3b	Choose appropriate manipulations for nan algebraic structure to make best use of structure (compound term contains factors)	$\boldsymbol{c - 3x = 4x}$ (linear structure) $\boldsymbol{a^2 - b^2}$ (difference of two squares: quadratic structure)	Find m from the following equation: $(m^2 - 2m)^2 = (m - 2)^2$

Example1: Structure Sense ability **1** (**SS1) -** Recognising structure in its simplest form.

Find $(2y + 4)^2$ *from the equation:* $2(2y + 4)^2 - 9 = (4y^2 + 16 + 16y) + 5$)

This question required a student to recognise the linear structure in the complex algebraic stricter where the term $(2y + 4)^2$ has to be determined its value. The students will solve the equation in a linear form without determining the value of y. If a student expands the bracket to find y, then the student has no structure sense ability, he/she is coded **PS** for procedural Strategy. In addition, if a student can name the simple structure in the equation, he/she has structure sense ability in recognising simple structure, if not, the subject is code **PS**, which implies that he/she lacks **SS1**. In this case procedural strategies **(PS)** are the methods that the student has been taught in class by the teacher or that is commonly used in the textbook.

Example 2: Structure Sense Ability 2 (**SS2) -** Deal with compound term as a single entity

Solve the equation: $(2m + 5)(m - 3) = (m - 3)$

A student who first expands/removes the brackets on the left side of the equation like; $2m(m - 3) + 5(m - 3) = (m - 3)$ is coded **PS** – procedural strategy, whilst a student who is able to deal with the term $(m - 3)$ as a single entity by first subtracting the bracket $(m - 3)$ from both sides of the equation and then apply the associative property of algebraic operation is coded **SS2.**

$$(2m + 5)(m - 3) = (m - 3)$$
$$(2m + 5)(m - 3) - (m - 3) = (m - 3) - (m - 3),$$
$$(m - 3)[(2m + 5) - 1] = 0$$
$$(m - 3) = 0 \ \ or \ \ [(2m + 5) - 1] = 0$$
$$m = 3 \quad or \quad 2m + 5 = 1$$
$$2m = 1 - 5$$
$$2m = -4$$
$$Hence\ m = 3 \quad or \quad m = -2$$

A student who uses the quadratic formulae to solve such an equation is also coded **PS** as procedural strategy.

Example: Structure Sense Ability 3- **SS3a**

Evaluate without using a calculator: $9745^2 - 9747 \times 9743$

A students who shows SS3a would probably manipulate the above numerical structure is the following manner: Let $a = 9745$ then $9745^2 - 9747 \times 9743$ can be written as $a^2 - (a+2)(a-2)$

$$9745^2 - 9747 \times 9743 \equiv a^2 - (a+2)(a-2)$$

$$\begin{aligned} 9745^2 - 9747 \times 9743 \quad &\equiv a^2 - (a^2 + 2a - 2a - 4) \\ &= 4 \end{aligned}$$

Students who multiply every value with the algorithmic method are said to have no structure sense ability in manipulating numerical structures, hence the code given is **PS**

Example 3: Item for Structure Sense 3b (**SS3b) -** Choosing apprpiate algebraic manipulation to make best use of structure.

A student who shows Structure sense 3b SS3b would probably be able to manipulate a complex structure into simpler units. In the case of the above example, a student makes use of his/her basic knowledge of the concept of difference of two squares to attack the problem.

Find the value of $\boldsymbol{m}$ *from the following equation:* $(m^2 - 2m)^2 = (m-2)^2$

Solution:

$$(m^2 - 2m)^2 = (m-2)^2$$
$$[m(m-2)]^2 = (m-2)^2$$
$$m^2(m-2)^2 = (m-2)^2$$
$$m^2(m-2)^2 - (m-2)^2 = (m-2)^2 - (m-2)^2$$
$$(m-2)^2(m^2-1) = 0$$
$$(m-2)(m-2)(m+1)(m-1) = 0$$

Therefore: $m = 2$ or $m = 2$ or $m = -1$ or $m = 1$

A students who solves this equation by expanding one or both sides of the

equation, is said to have no structure sense ability to manipulate algebraic complex structures to familiar simple structures, so this kind of activity is coded **PS** for procedural strategy.

3.7.3 Analysing Flexibility

Students' algebraic structure sense flexibility was seen in the way students were flexible in solving the equation. A student who showed algebraic structure sense flexibility was the one who provided the answers through multiple (diverse) ways **(**coded **FS1)** or in unique ways which has not been taught before **(**coded **FS2)** or which has not been taught by the teacher in class, otherwise it's a **1** (standing for one method used)**.** Again this criterion of classifying flexibility was adopted from the definition of mathematical flexibility which is given by Xu et al. (2017) as well as NCTM (2012). However, the coding itself for students' flexibility was a self-invented one by the researcher.

3.7.4 Analysing Difficulties

Difficulties were described in words as they appeared. Difficulties were viewed in three categories; 1) Computational and omission errors, 2) Misunderstanding of Variables and equivalence relations 3) Errors in algebraic operations which include the use of brackets, associative, distributive and commutative properties and 4) Emotional difficulties. Indicators of difficulties are adopted and modified from several researchers on students' difficulties in algebra (Knuth, 2005, Booth et al., 2014, Weinberg, Dresen, & Slater, 2016). Categories of difficulties are summarized in **Table 3**.

Table 3.3
Signs for Algebraic Difficulties

Category Number	Category of Difficulties	Sub-category/Description of a difficulties
1	Computational Errors and Omissions	-Errors in multiplication, addition, subtraction, division of integers. -Use of calculators against the explicit instruction -Errors in integer operation -Omissions of negative signs, omissions of brackets -Questions quitted/not attempted

2	Understanding of variables and equivalence relations.	-Misrepresentation of the " = " sign -misunderstanding equivalence relations -Unable identify the nature/structure of equations (to state whether it is linear or quadratic). -Not using/understanding the additive/multiplicative inverses in solving equations.
3	Algebraic manipulations	-Errors in the use of associative and distributive property. -Errors in order of operation and in power/indices multiplication -Errors in common factors
4	Emotional Difficulties	-Unable to explain working processes -Unwillingness to cooperate in the interview session

3.8 Research Ethics

The major ethical aspects that were taken in this research were anonymity, confidentiality and morality, review of institutional requirements, communication and non-plagiarism. Ethics are research guidelines that encourage responsible research practices and ensure the protection of human research participants (Suter, 2012 pg. 97). The researcher has maintain anonymity by not revealing the names of the research subjects or respondents in-order to protect their privacy (O'Reilly & Kiyimba, 2015). The names of the school that has been the place of research and the names of the students on the answer sheets have been concealed. The data collected is kept confidentially by the researcher and only results will be reported and published if possible.

The researcher did her best to be moral, polite and formally dressed during the time of visiting the research field, (the school). The researcher was proper and polite language and neutral facial expressions during data collection, According to O'Reilly et al. (2013), subjects of this age (13-18years) have capacity to understand and "decide for themselves if they want to engage in the research or not especially when it comes to issues of academic and social relationship. It is better to keep them engaged…" (O'Reilly et al., 2013 pg. 48). From this view, the researcher tried to her level best to

make the data collection process interesting for subjects to keep them engaged.

Subjects were informed about the purpose of research, the date and time that they were going to write the test and to sit for interviews. The researcher also emphasized to the students that the results of the test were not going to contribute to their end of semester grade since the test was written towards the end of semester 1 of grade 10. The research subjects and the mathematics teachers signed consent forms as indications that they are willing to participate in the research study (See Appendices for Student and Teacher Consent forms). Participants were told that they are free to withdraw from the study if they feel like doing so and were not coerced to participate in the study. The researcher tried her best to minimize the normal order of the business by collecting data during students' free sessions. The researcher also made appointment with the teacher every time she was going for data collection. Participants (research subjects and the mathematics teacher) received some tokens of appreciation for their effort in the current study.

The researcher has obtained a formal introductory letter from Universitas Pendidikan Indonesia (the researcher's institution) which was taken to school to seek permission to conduct the research at the school. The whole process in this research process is and will be free of plagiarism. The research is an original of the researcher herself and if any ideas and materials were obtained from other sources, then it is correctly cited or referenced. During writing and reporting results, the researcher followed the Institutional style (UPI Penaturan Teknis Menulis, 2015) and international referencing style that are recommended by the institution review board.