DESAIN DIDAKTIS TOPIK FUNGSI BERDASARKAN KAJIAN KEMAMPUAN *FUNCTIONAL THINKING* SISWA DI SEKOLAH MENENGAH PERTAMA

DISERTASI

diajukan untuk memenuhi sebagian syarat untuk memperoleh gelar doktor
Pendidikan Matematika

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PROGRAM STUDI DOKTOR PENDIDIKAN MATEMATIKA
FAKULTAS PENDIDIKAN MATEMATIKA DAN ILMU PENGETAHUAN ALAM
UNIVERSITAS PENDIDIKAN INDONESIA
2025

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DESAIN DIDAKTIS TOPIK FUNGSI
BERDASARKAN KAJIAN KEMAMPUAN *FUNCTIONAL THINKING*
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Nadya Syifa Utami

ABSTRAK

Nadya Syifa Utami (2217307). Desain Didaktis Topik Fungsi berdasarkan Kajian Kemampuan *Functional Thinking* Siswa di Sekolah Menengah Pertama.

Pentingnya kemampuan *functional thinking* dalam membangun konsep fungsi berbanding terbalik dengan kesulitan yang dialami siswa dalam menyelesaikan masalah yang menuntut kemampuan ini, khususnya ketika siswa melakukan generalisasi simbolik pada pola yang diberikan. Penelitian ini bertujuan untuk menghasilkan desain didaktis topik fungsi berdasarkan hasil kajian kemampuan *functional thinking* siswa. Penelitian kualitatif ini menggunakan pendekatan fenomenologi hermeneutik. Desain penelitian yang digunakan adalah *Didactical Design Research* yang memuat tiga tahap analisis: prospektif (penyusunan desain), metapedadidaktik (implementasi desain), dan retrospektif (refleksi dan evaluasi desain). Adapun partisipan 40 siswa kelas 9 SMP dan seorang guru matematika (identifikasi *learning obstacle*) dan 21 siswa kelas 8 SMP (implementasi desain didaktis). Pengumpulan data dalam penelitian ini dilakukan melalui studi dokumentasi, observasi, wawancara, tes, dan rekaman audio dan video. Hasil penelitian ini menunjukkan bahwa: (1) terdapat perbedaan makna pada fungsi antara *scholarly knowledge* dengan *knowledge to be taught* dan *taught knowledge*, sehingga *learned knowledge* siswa pada fungsi sebagai *changing quantities* banyak mengalami masalah; (2) dua jenis *learning obstacle* siswa teridentifikasi: *epistemological obstacle*, yaitu keterbatasan pemahaman siswa tentang fungsi sebagai hubungan antar kuantitas, sementara *didactical obstacle* terjadi ketika sajian materi baik dalam buku teks maupun desain pembelajaran guru yang tidak memfasilitasi siswa memahami rumus dan grafik fungsi melalui konteks nyata; (3) *hypothetical learning trajectory* disusun untuk mengakomodasi alur berpikir siswa mulai dari pengertian fungsi, fungsi linear, hingga grafik fungsi linear melalui pemanfaatan kemampuan *functional thinking*; (4) desain didaktis hipotetik topik fungsi disusun berdasarkan *theory of didactical situation* yang memuat empat situasi yaitu aksi, formulasi, validasi, dan institusionalisasi; (5) Implementasi desain menunjukkan bahwa interaksi siswa dengan lingkungan pembelajaran (*millieu*) dapat menciptakan situasi didaktis yang mendukung pembangunan pengetahuan fungsi; (6) meskipun hasil implementasi desain memberikan hasil yang positif, diperlukan perbaikan khususnya pada *series of tasks* tentang tingkat perubahan fungsi dan grafik fungsi linear pada desain didaktis empiris topik fungsi. Secara keseluruhan, desain didaktis ini dapat menjadi salah satu desain alternatif dalam melaksanakan pembelajaran topik fungsi di SMP.

Kata kunci: Desain didaktis, Fungsi, *Functional Thinking*, *Learning Obstacle*

ABSTRACT

Nadya Syifa Utami (2217307). Didactical Desain in the Topic of Functions based on the Study of Students' Functional Thinking Ability in Junior High School.

The importance of functional thinking in building the concept of functions is inversely related to the difficulties faced by students in solving problems that require this ability, especially when students generalize symbolic representations of the given patterns. This study aims to construct a didactical design for the topic of functions based on the study of students' functional thinking abilities. This qualitative research used a phenomenological hermeneutic approach. The research design employed was Didactical Design Research, which consisted of three stages of analysis: prospective (constructed the design), metapedagogical (implemented the design), and retrospective (reflected and evaluated the design). The participants included 40 ninth-grade students and a mathematics teacher (to identify learning obstacles), as well as 21 eighth-grade students (for the implementation of the didactical design). Data collection in this study was carried out through documentation study, observations, interviews, tests, and audio and video recordings. The findings of this study indicate that: (1) there is a difference in the meaning of functions between scholarly knowledge, knowledge to be taught, and taught knowledge, leading to problems in students' learned knowledge of functions as changing quantities; (2) two types of student learning obstacles were identified: epistemological obstacles, which occur due to students' limited understanding of functions as relationships between quantities, and didactical obstacles, which arise when the tasks presented in textbooks or by the teacher's teaching design does not facilitate students in understanding the formulas and graphs of functions through real-life contexts; (3) a hypothetical learning trajectory was developed to accommodate the students' thought processes from understanding functions, linear functions, to the graph of linear functions, utilizing functional thinking; (4) the hypothetical didactical design for the topic of functions was structured based on the theory of didactical situations, which includes four situations: action, formulation, validation, and institutionalization; (5) the implementation of the design shows that interactions between students and the learning environment (millieu) can create didactical situations that support the development of function knowledge; (6) although the results of the implementation were positive, improvements are needed, particularly in the series of tasks on the rate of change of functions and the graph of linear functions in the empirical didactical design of the topic of functions. Overall, this didactical design can serve as one of the alternative designs for teaching the topic of functions in junior high schools.

Kata kunci: Didactical Design, Functions, Functional Thinking, Learning Obstacle

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DAFTAR GAMBAR

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