

**PENGARUH MODEL *PROBLEM-BASED LEARNING* MENGGUNAKAN
PENDEKATAN *OPEN-ENDED* TERHADAP KEMAMPUAN
PENALARAN MATEMATIS SISWA SD**

TESIS

diajukan untuk memenuhi sebagian syarat untuk memperoleh gelar magister
Pendidikan Dasar



Oleh

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**PROGRAM STUDI PENDIDIKAN DASAR
FAKULTAS ILMU PENDIDIKAN
UNIVERSITAS PENDIDIKAN INDONESIA**

2025

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S.Pd. Universitas Pendidikan Indonesia, 2017

Sebuah Tesis yang diajukan untuk memenuhi salah satu syarat memperoleh gelar Magister
Pendidikan (M.Pd.) pada Fakultas Ilmu Pendidikan

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Universitas Pendidikan Indonesia

Februari 2025

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ABSTRAK

Penelitian ini bertujuan untuk menganalisis dan mendeskripsikan keefektifan pengaruh *Problem-Based Learning* (PBL) menggunakan pendekatan *open-ended* terhadap kemampuan penalaran matematis siswa sekolah dasar. Metode penelitian yang digunakan adalah kuantitatif dengan tipe kuasi eksperimen. Desain penelitian yang digunakan adalah *one group pre-test post-test group design* dan *pre-test post-test control group design*. Kelas kontrol mengimplementasikan pembelajaran langsung dan kelas eksperimen mengimplementasikan model PBL menggunakan pendekatan *open-ended*. Sampel yang digunakan pada penelitian adalah kemampuan penalaran matematis siswa kelas IV SD salah satu SD negeri di Kota Bandung sebanyak 24 siswa di kelas kontrol dan 24 siswa di kelas eksperimen. Instrumen yang digunakan adalah tes penalaran matematis materi pecahan. Hasil penelitian menunjukkan 1) rata-rata perolehan siswa yang memperoleh pembelajaran PBL menggunakan pendekatan *open-ended* lebih tinggi dari pada siswa yang memperoleh pembelajaran langsung; 2) PBL dan pembelajaran langsung berpengaruh secara signifikan terhadap kemampuan penalaran matematis, dan 3) peningkatan kemampuan penalaran spasial siswa yang memperoleh pembelajaran PBL dan pembelajaran langsung berada pada kategori sedang, 4) dan terdapat perbedaan pengaruh model PBL menggunakan pendekatan *open-ended* dan pembelajaran langsung pada perolehan dan peningkatan kemampuan penalaran matematis siswa sekolah dasar.

Kata kunci: *Problem-Based Learning*, *Open-Ended*, Kemampuan Penalaran Matematis, Sekolah Dasar

***THE EFFECT OF PROBLEM-BASED LEARNING MODEL USING OPEN-
ENDED APPROACH ON MATHEMATICAL REASONING ABILITY OF
ELEMENTARY SCHOOL STUDENTS***

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ABSTRACT

This study aims to analyze and describe the effectiveness of Problem-Based Learning (PBL) using an open-ended approach on mathematical reasoning ability of elementary school students. The research method used is quantitative with quasi-experimental type. The research design used was one group pre-test post-test group design and pre-test post-test control group design. The control class implemented direct learning and the experimental class implemented PBL model using open-ended approach. The sample used in the study was the mathematical reasoning ability of fourth grade students of one of the public elementary schools in Bandung consisting of 24 students in the control class and 24 students in the experimental class. The instrument used was a mathematical reasoning test on fraction material. The results showed 1) the average score of students who obtained PBL learning using an open-ended approach was higher than that of students who obtained direct learning; 2) PBL and direct learning have a significant effect on mathematical reasoning ability, and 3) the improvement of spatial reasoning ability of students who get PBL learning and direct learning is in the medium category, 4) and there is a difference in the effect of PBL model using open-ended approach and direct learning on the obtaining and improving mathematical reasoning ability of elementary school students.

Keywords: *Problem-Based Learning, open-ended, Mathematical Reasoning Ability, Fractions*

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