

**DESAIN DIDAKTIS UNTUK PROSES GENERALISASI:  
FOKUS MATERI POLA BILANGAN**

**TESIS**

Diajukan untuk memenuhi salah satu syarat untuk memperoleh gelar  
Magister Pendidikan Matematika



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## **LEMBAR HAK CIPTA**

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untuk memperoleh gelar Magister Pendidikan (M.Pd.)  
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## LEMBAR PENGESAHAN TESIS

### DESAIN DIDAKTIS UNTUK PROSES GENERALISASI: FOKUS MATERI POLA BILANGAN

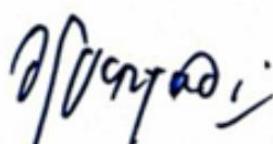
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## ABSTRAK

**Nyla Farhatul Maula** (2217017). Desain Didaktis untuk Proses Generalisasi: Fokus Materi Pola Bilangan.

Penelitian ini bertujuan untuk mengidentifikasi hambatan belajar dalam proses generalisasi pola bilangan pada siswa dan mengembangkan desain didaktis hipotesis untuk mengatasi hambatan tersebut. Hambatan belajar ditinjau melalui tiga kategori utama: ontogenik, didaktik, dan epistemologi. Studi ini melibatkan analisis terhadap desain pembelajaran yang ada, serta implementasi instrumen asesmen untuk mengidentifikasi hambatan yang dialami siswa. Temuan menunjukkan bahwa hambatan ontogenik meliputi kesulitan siswa dalam memahami konsep akibat keterbatasan mental dan kesiapan belajar. Hambatan didaktik diidentifikasi terkait dengan penyajian materi dan urutan pembelajaran yang kurang sesuai dengan alur pikir siswa, sementara hambatan epistemologi mencakup keterbatasan pemahaman siswa dalam mengaplikasikan konsep pada konteks baru. Berdasarkan hasil analisis, penelitian ini mengusulkan lintasan belajar yang disesuaikan dengan kebutuhan siswa untuk mendukung proses generalisasi pola bilangan. Desain didaktis yang dihasilkan diharapkan dapat memfasilitasi siswa dalam mengembangkan pemahaman pola bilangan secara bertahap dan mendalam. Temuan ini dapat memberikan kontribusi pada pengembangan desain didaktis pada pembelajaran matematika yang lebih efektif dalam mendukung proses generalisasi dan mengatasi hambatan belajar.

Kata kunci:

Desain Didaktis, Generalisasi, Hambatan Belajar, Pola Bilangan, Lintasan Belajar

## ABSTRACT

**Nyla Farhatul Maula** (2217017). Didactic Design for Generalization Process: Focus on Number Pattern.

This study aims to identify learning obstacles in students' generalization of number patterns and to develop a hypothetical didactic design to address these obstacles. Learning obstacles are reviewed through three main categories: ontogenetic, didactic, and epistemological. The study involves an analysis of existing instructional designs and the implementation of assessment instruments to identify obstacles encountered by students. Findings indicate that ontogenetic obstacles include difficulties in understanding concepts due to mental limitations and readiness to learn. Didactic obstacles are identified as related to the presentation of material and the instructional sequence, which are not fully aligned with students' thought processes. Meanwhile, epistemological obstacles encompass students' limited understanding in applying concepts to new contexts. Based on the analysis, this study proposes a learning trajectory tailored to students' needs to support the generalization process of number patterns. The resulting didactic design is expected to facilitate students in developing a gradual and in-depth understanding of number patterns. These findings contribute to the development of a more effective didactic design in mathematics education, supporting generalization processes and overcoming learning obstacles.

Keywords:

Didactic Design, Generalization, Learning Obstacles, Number Patterns, Learning Trajectory

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