

**DESAIN DIDAKTIS MATERI PERBANDINGAN YANG TERKAIT  
DENGAN KEMAMPUAN *PROPORTIONAL REASONING* PADA SISWA  
SEKOLAH MENENGAH PERTAMA**

TESIS

diajukan untuk memenuhi salah satu syarat untuk memperoleh gelar magister  
pada Program Studi Pendidikan Matematika



oleh

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## ABSTRAK

Ifa Fathiyah. (2113128). **Desain Didaktis Materi Perbandingan yang Terkait dengan Kemampuan Proportional Reasoning pada Siswa Sekolah Menengah Pertama.**

Kemampuan *proportional reasoning* merupakan kemampuan untuk bernalar menggunakan hubungan perkalian pada sebuah situasi yang melibatkan dua rasio yang *equal*. Kemampuan ini penting untuk dimiliki siswa karena menjadi dasar untuk memahami beberapa konsep seperti aljabar, geometri, dan statistika. Meskipun begitu, penelitian-penelitian terdahulu mengungkapkan bahwa kemampuan *proportional reasoning* siswa masih lemah. Oleh karena itu, penelitian ini bertujuan untuk mengembangkan desain didaktis materi perbandingan yang dikaitkan juga pada pengembangan kemampuan *proportional reasoning*. Materi tersebut dipilih karena merupakan konsep dasar untuk memahami konsep matematika selanjutnya. Desain yang dihasilkan diharapkan dapat meminimalisir hambatan siswa dalam memahami materi perbandingan serta mengembangkan kemampuan *proportional reasoning*-nya. Berdasarkan tujuan penelitian tersebut, penelitian ini merupakan penelitian kualitatif yang juga menjadi bagian dari *Didactical Design Research* (DDR). Subjek penelitian ini adalah 49 siswa kelas VIII dari salah satu SMP di Kota Bandung yang kemudian dipilih 9 siswa untuk dikaji lebih dalam kemampuan *proportional reasoning* dan *learning obstacle*-nya untuk menjadi bahan pertimbangan pembuatan *hypothetical learning trajectory* (HLT). HLT tersebut kemudian menjadi acuan pengembangan desain didaktis dalam penelitian ini. Temuan penelitian menunjukkan bahwa tidak ada satupun siswa yang memenuhi seluruh indikator kemampuan *proportional reasoning* dan menunjukkan adanya *ontogenic obstacle*, *epistemological obstacle*, serta potensi *didactical obstacle* yang dialami siswa. HLT yang dikembangkan diawali dari pembelajaran konsep rasio kemudian pembelajaran konsep perbandingan senilai dan konsep perbandingan berbalik nilai. Desain didaktis rekomendasi yang dikembangkan mengarahkan siswa untuk bisa menggunakan konsep perbandingan dalam menyelesaikan suatu masalah sekaligus mengembangkan kemampuan *proportional reasoning*-nya. Desain tersebut juga mencakup empat tahapan situasi didaktis, yaitu situasi aksi, formulasi, validasi, dan institusionalisasi.

## ABSTRACT

Ifa Fathiyah. (2113128). **Didactical Design of Proportion Material Related to Proportional Reasoning Ability in Junior High School.**

Proportional reasoning ability is the ability to reason in a situation involving two equal ratios by using multiplication relationships. Students must possess this ability because it serves as the foundation for learning numerous concepts, including statistics, geometry, and algebra. However, prior research demonstrated that students appear to have limited ability in proportional reasoning. This study therefore attempts to develop a didactical design for proportion material that is also related to the development of proportional reasoning ability. It is anticipated that the design will reduce obstacles in understanding the proportion material and enhance students' proportional reasoning ability. Consequently, this research is qualitative research which is also part of Didactical Design Research (DDR). The participants in this research were 49 eighth-grade students from a junior high school in Bandung City. Their proportional reasoning ability and learning obstacles, which were taken into account when developing a hypothetical learning trajectory (HLT), were evaluated. The research findings indicated that all students failed to satisfy all the indicators of proportional reasoning ability and showed the existence of ontogenic obstacles, epistemological obstacles, and potential didactical obstacles experienced by students. The HLT developed started from learning the concept of ratio, followed by learning the direct proportion and inverse proportion concept. The didactical design developed based on the HLT directs students to be able to use the concept of proportion in solving a problem while developing their proportional reasoning ability and includes four stages of didactical situations, namely situations of action, formulation, validation, and institutionalization.

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